

Mathematics

Advanced GCE A2 7890 - 2

Advanced Subsidiary GCE AS 3890 - 2

Mark Schemes for the Units

June 2008

3890-2/7890-2/MS/R/08

OCR (Oxford, Cambridge and RSA Examinations) is a unitary awarding body, established by the University of Cambridge Local Examinations Syndicate and the RSA Examinations Board in January 1998. OCR provides a full range of GCSE, A level, GNVQ, Key Skills and other qualifications for schools and colleges in the United Kingdom, including those previously provided by MEG and OCEAC. It is also responsible for developing new syllabuses to meet national requirements and the needs of students and teachers.

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the syllabus content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

OCR will not enter into any discussion or correspondence in connection with this Report.

© OCR 2008

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

CONTENTS

Advanced GCE Mathematics (7890)
Advanced GCE Pure Mathematics (7891)
Advanced GCE Further Mathematics (7892)

Advanced Subsidiary GCE Mathematics (3890)
Advanced Subsidiary GCE Pure Mathematics (3891)
Advanced Subsidiary GCE Further Mathematics (3892)

MARK SCHEMES FOR THE UNITS

Unit/ContentPage

4721 Core Mathematics 1	1
4722 Core Mathematics 2	6
4723 Core Mathematics 3	10
4724 Core Mathematics 4	13
4725 Further Pure Mathematics 1	17
4726 Further Pure Mathematics 2	21
4727 Further Pure Mathematics 3	24
4728 Mechanics 1	31
4729 Mechanics 2	33
4730 Mechanics 3	35
4731 Mechanics 4	38
4732 Probability & Statistics 1	42
4733 Probability & Statistics 2	46
4734 Probability & Statistics 3	487
4735 Statistics 4	510
4736 Decision Mathematics 1	543
4737 Decision Mathematics 2	598
Grade Thresholds	643

4721 Core Mathematics 1

1 (i)	$n = -2$	B1	
		<input type="text" value="1"/>	
(ii)	$n = 3$	B1	
		<input type="text" value="1"/>	
(iii)		M1	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $(4^3)^{\frac{1}{2}}$ or $4 \times \sqrt{4}$ with brackets correct if used
	$n = \frac{3}{2}$	A1	
		<input type="text" value="2"/>	
2 (i)	$y = (x - 2)^2$	M1	$y = (x \pm 2)^2$
		A1	
		<input type="text" value="2"/>	
(ii)	$y = -(x^3 - 4)$	B1	oe
		<input type="text" value="1"/>	
3 (i)	$\sqrt{2 \times 100} = 10\sqrt{2}$	B1	
		<input type="text" value="1"/>	
(ii)	$\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	B1	
		<input type="text" value="1"/>	
(iii)	$10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	M1	Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$
		A1	
		<input type="text" value="2"/>	
4	$y = x^{\frac{1}{2}}$ $2y^2 - 7y + 3 = 0$ $(2y - 1)(y - 3) = 0$ $y = \frac{1}{2}, y = 3$ $x = \frac{1}{4}, x = 9$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$
		M1dep	Correct method to solve a quadratic
		A1	
		M1	Attempt to square to obtain x
		A1	
		SR	If first M1 not gained and 3 and $\frac{1}{2}$ given as final answers, award B1
		<input type="text" value="5"/>	

5

$$\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$$

$$= 4\left(\frac{1}{\sqrt{9}}\right) + 1$$

$$\frac{dy}{dx} = \frac{7}{3}$$

M1 Attempt to differentiate

A1 $kx^{-\frac{1}{2}}$

A1

M1 Correct substitution of $x = 9$ into their

A1 $\frac{7}{3}$ only

5

6 (i) $(x-5)(x+2)(x+5)$

$$= (x^2 - 3x - 10)(x+5)$$

$$= x^3 + 2x^2 - 25x - 50$$

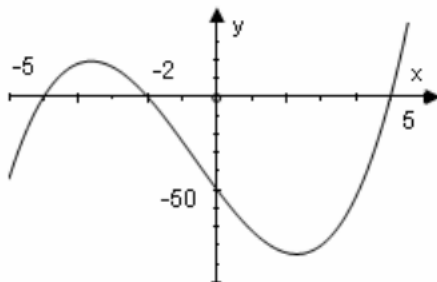
B1 $x^2 - 3x - 10$ or $x^2 + 7x + 10$ or $x^2 - 25$ seen

M1 Attempt to multiply a quadratic by a linear factor

A1

3

(ii)



B1 +ve cubic with 3 roots (not 3 line segments)

B1✓ (0, -50) labelled or indicated on y-axis

B1 (-5, 0), (-2, 0), (5, 0) labelled or indicated on x-axis and no other x- intercepts

3

7 (i) $8 < 3x - 2 < 11$

$$10 < 3x < 13$$

$$\frac{10}{3} < x < \frac{13}{3}$$

M1 2 equations or inequalities both dealing with all 3 terms resulting in $a < kx < b$

A1 10 and 13 seen

A1

3

(ii) $x(x+2) \geq 0$

M1 Correct method to solve a quadratic

A1 0, -2

M1 Correct method to solve inequality

A1

4

$$x \geq 0, x \leq -2$$

8 (i) $\frac{dy}{dx} = 3x^2 - 2kx + 1$	B1 One term correct B1 Fully correct <div style="border: 1px solid black; padding: 2px; display: inline-block;">2</div>
(ii) $3x^2 - 2kx + 1 = 0$ when $x = 1$ $3 - 2k + 1 = 0$ $k = 2$	M1 their $\frac{dy}{dx} = 0$ soi M1 $x = 1$ substituted into their $\frac{dy}{dx} = 0$ A1 ✓ <div style="border: 1px solid black; padding: 2px; display: inline-block;">3</div>
(iii) $\frac{d^2y}{dx^2} = 6x - 4$ When $x = 1$, $\frac{d^2y}{dx^2} > 0 \therefore$ min pt	M1 Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign A1 States minimum CWO <div style="border: 1px solid black; padding: 2px; display: inline-block;">2</div>
(iv) $3x^2 - 4x + 1 = 0$ $(3x - 1)(x - 1) = 0$ $x = \frac{1}{3}, x = 1$ $x = \frac{1}{3}$	M1 their $\frac{dy}{dx} = 0$ M1 correct method to solve 3-term quadratic A1 WWW at any stage <div style="border: 1px solid black; padding: 2px; display: inline-block;">3</div>

<p>9 (i)</p> $(x-2)^2 + (y-1)^2 = 100$ $x^2 + y^2 - 4x - 2y - 95 = 0$	<p>B1 $(x-2)^2$ and $(y-1)^2$ seen</p> <p>B1 $(x \pm 2)^2 + (y \pm 1)^2 = 100$</p> <p>B1 correct form</p> <p>3</p>
<p>(ii)</p> $(5-2)^2 + (k-1)^2 = 100$ $(k-1)^2 = 91 \quad \text{or} \quad k^2 - 2k - 90 = 0$ $k = 1 + \sqrt{91}$	<p>M1 $x = 5$ substituted into their equation</p> <p>A1 correct, simplified quadratic in k (or y) obtained</p> <p>A1 cao</p> <p>3</p>
<p>(iii) distance from $(-3, 9)$ to $(2, 1)$</p> $= \sqrt{(2 - (-3))^2 + (1 - 9)^2}$ $= \sqrt{25 + 64}$ $= \sqrt{89}$ $\sqrt{89} < 10 \quad \text{so point is inside}$	<p>M1 Uses $(x_2 - x_1)^2 + (y_2 - y_1)^2$</p> <p>A1</p> <p>B1 compares their distance with 10 and makes consistent conclusion</p> <p>3</p>
<p>(iv) gradient of radius = $\frac{9-1}{8-2}$</p> $= \frac{4}{3}$ <p>gradient of tangent = $-\frac{3}{4}$</p> $y - 9 = -\frac{3}{4}(x - 8)$ $y - 9 = -\frac{3}{4}x + 6$ $y = -\frac{3}{4}x + 15$	<p>M1 uses $\frac{y_2 - y_1}{x_2 - x_1}$</p> <p>A1 oe</p> <p>B1✓ oe</p> <p>M1 correct equation of straight line through $(8, 9)$, any non-zero gradient</p> <p>A1 oe 3 term equation</p> <p>5</p>

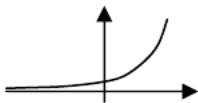
<p>10 (i) $2(x^2 - 3x) + 11$ $= 2\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$ $= 2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$</p>	<p>B1 $p = 2$ B1 $q = -\frac{3}{2}$ M1 $r = 11 - 2q^2$ or $\frac{11}{2} - q^2$ A1 $r = \frac{13}{2}$</p> <p style="text-align: center;">4</p>
<p>(ii) $\left(\frac{3}{2}, \frac{13}{2}\right)$</p>	<p>B1√ B1√ 2</p>
<p>(iii) $36 - 4 \times 2 \times 11$ $= -52$</p>	<p>M1 uses $b^2 - 4ac$ A1 2</p>
<p>(iv) 0 real roots</p>	<p>B1 cao 1</p>
<p>(v) $2x^2 - 6x + 11 = 14 - 7x$ $2x^2 + x - 3 = 0$ $(2x + 3)(x - 1) = 0$ $x = -\frac{3}{2}, x = 1$ $y = \frac{49}{2}, y = 7$</p>	<p>M1* substitute for x/y or attempt to get an equation in 1 variable only A1 obtain correct 3 term quadratic M1dep correct method to solve 3 term quadratic A1 A1</p> <p>SR If A0 A0, one correct pair of values, spotted or from correct factorisation www B1</p> <p style="text-align: center;">5</p>

4722 Core Mathematics 2

<p>1</p> $(2 - 3x)^6 = 2^6 + 6 \cdot 2^5 \cdot (-3x) + 15 \cdot 2^4 \cdot (-3x)^2 + 20 \cdot 2^3 \cdot (-3x)^3 + 15 \cdot 2^2 \cdot (-3x)^4 + 6 \cdot 2 \cdot (-3x)^5 + (-3x)^6$ $= 64 - 576x + 2160x^2 - 5400x^3 + 3780x^4 - 1764x^5 + 729x^6$ <p>OR</p>	<p>M1 Attempt (at least) first two terms - product of binomial coefficient and powers of 2 and (-3x)</p> <p>A1 Obtain $64 - 576x$</p> <p>M1 Attempt third term - binomial coefficient and powers of 2 and (-3x)</p> <p>A1 Obtain $2160x^2$</p> <p>M1 Attempt expansion involving all 6 brackets</p> <p>A1 Obtain 64</p> <p>A1 Obtain $-576x$</p> <p>A1 Obtain $2160x^2$</p>
<p>SR if the expansion is attempted in descending order, and the required terms are never seen, then B1 B1 B1 for $4860x^4, -2916x^5, 729x^6$</p>	<p style="text-align: center;">4</p>
<p>2 (i)</p> $u_2 = \frac{2}{3}$ $u_3 = \frac{-1}{2}$ $u_4 = 3$	<p>B1 Obtain correct u_2</p> <p>B1 Obtain correct u_3 from their u_2</p> <p>B1 Obtain correct u_4 from their u_3</p> <p style="text-align: center;">3</p>
<p>(ii) sequence is periodic / cyclic / repeating</p>	<p>B1 Any equivalent comment</p> <p style="text-align: center;">1</p>
<p>3 (i)</p> $\frac{1}{2} \times 8^2 \times \theta = 48$ <p>Hence $\theta = 1.5$ radians</p>	<p>M1 State or imply $(\frac{1}{2}) 8^2 \theta = 48$</p> <p>A1 Obtain $\theta = 1.5$ (or 0.477π), or equiv</p> <p style="text-align: center;">2</p>
<p>(ii)</p> $\text{area} = 48 - \frac{1}{2} \times 8^2 \times \sin 1.5$ $= 48 - 31.9$ $= 16.1$	<p>M1* Attempt area of Δ using $(\frac{1}{2}) 8^2 \sin \theta$</p> <p>M1d* Attempt $48 - \text{area of } \Delta$</p> <p>A1 Obtain 16.1 cm^2</p> <p style="text-align: center;">3</p>
<p>4 (i)</p> $f(3) = 27a - 36 - 21a + 12 = 0$ $6a = 24$ $a = 4$ <p>OR</p>	<p>M1* Attempt $f(3)$</p> <p>M1d* Equate attempt at $f(3)$ to 0 and attempt to solve</p> <p>A1 Obtain $a = 4$</p> <p>M1* Attempt complete division / matching coeffs</p> <p>M1d* Equate remainder to 0</p> <p>A1 Obtain $a = 4$</p> <p style="text-align: center;">3</p>
<p>(ii)</p> $f(-2) = -32 - 16 + 56 + 12$ $= 20$	<p>M1 Attempt $f(-2)$</p> <p>A1 Obtain 20 (or $6a - 4$, following their a)</p> <p style="text-align: center;">2</p>

<p>5 (i) $\int xdy = \int ((y-3)^2 - 2)dy$ $= \int (y^2 - 6y + 7)dy$ A.G. $3 + \sqrt{(2+2)} = 5, 3 + \sqrt{(14+2)} = 7$</p>	<p>B1 Show $x = y^2 - 6y + 7$ convincingly B1 State or imply that required area $= \int xdy$ B1 Use $x = 2, 14$ to show new limits of $y = 5, 7$ 3</p>
<p>(ii) $\left[\frac{1}{3}y^3 - 3y^2 + 7y \right]_5^{14}$ term $= ({}^{343}/_3 - 147 + 49) - ({}^{125}/_3 - 75 + 35)$ $= 16\frac{1}{3} - 1\frac{2}{3}$ $= 14\frac{2}{3}$</p>	<p>M1 Integration attempt, with at least one correct A1 All three terms correct M1 Attempt $F(7) - F(5)$ A1 Obtain $14\frac{2}{3}$, or exact equiv 4</p>
<p>6 (i) $ABC = 360 - (150 + 110) = 100^\circ$ A.G.</p>	<p>B1 Show convincingly that angle ABC is 100° 1</p>
<p>(ii) $CA^2 = 15^2 + 27^2 - 2 \times 15 \times 27 \times \cos 100^\circ$ $= 1094.655\dots$ $CA = 33.1$</p>	<p>M1 Attempt use of correct cosine rule A1 Obtain 33.1 km 2</p>
<p>(iii) $\frac{\sin C}{15} = \frac{\sin 100}{33.1}$ or $\frac{\sin A}{27} = \frac{\sin 100}{33.1}$ $C = 26.5^\circ$ $A = 53.5^\circ$ Hence bearing is 263°</p>	<p>M1 Attempt use of sine rule to find angle C or A (or equiv using cosine rule) A1 Correct unsimplified eqn, following their CA A1 Obtain $C = 26.5^\circ$ or $A = 53.5^\circ$ (allow 53.4°) A1 Obtain 263 or 264 (or $290^\circ -$ their angle $C / 210 +$ their angle A) 4</p>
<p>7 (a) $\int (x^5 - x^4 + 5x^3)dx$ $= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4 (+c)$</p>	<p>M1 Expand brackets and attempt integration, or other valid integration attempt A1 Obtain at least one correct term A1 Obtain a fully correct expression B1 For $+c$, and no \int or dx (can be given in (b)(i) if not given here) 4</p>
<p>(b) (i) $-6x^{-3} (+c)$</p>	<p>M1 Obtain integral of the form kx^{-3} A1 Obtain $-6x^{-3} (+c)$ 2</p>
<p>(ii) $\left[-6x^{-3} \right]_0^\infty$ $= \frac{3}{4}$</p>	<p>B1* State or imply that $F(\infty) = 0$ (for $kx^n, n < -1$) B1d* Obtain $\frac{3}{4}$ (or equiv) 2</p>

8 (i)



- M1** Attempt sketch of exponential graph (1st quad)
- if seen in 2nd quad must be approx correct
- A1** Correct graph in both quadrants
- B1** State or imply (0, 2) only

3

(ii) $8^x = 2 \times 3^x$
 $\log_2 8^x = \log_2 (2 \times 3^x)$

$$x \log_2 8 = \log_2 2 + x \log_2 3$$

$$3x = 1 + x \log_2 3$$

$$x(3 - \log_2 3) = 1, \text{ hence } x = \frac{1}{3 - \log_2 3} \text{ A.G.}$$

- M1** Form equation in x and take logs (to any consistent base, or no base) – could use \log_8
- M1** Use $\log a^b = b \log a$
- M1** Use $\log ab = \log a + \log b$, or equiv with $\log^{a/b}$
- M1** Use $\log_2 8 = 3$
- A1** Show given answer correctly

OR $8^x = 2 \times 3^x$

$$2^{3x} = 2 \times 3^x$$

$$2^{(3x-1)} = 3^x$$

$$\log_2 2^{(3x-1)} = \log_2 3^x$$

$$(3x-1) \log_2 2 = x \log_2 3$$

$$x(3 - \log_2 3) = 1, \text{ hence } x = \frac{1}{3 - \log_2 3} \text{ A.G.}$$

- M1** Use $8^x = 2^{3x}$
- M1** Attempt to rearrange equation to $2^k = 3^x$
- M1** Take logs (to any base)
- M1** Use $\log a^b = b \log a$
- A1** Show given answer correctly

5

9 (a) (i) $2 \sin x \frac{\sin x}{\cos x} - 5 = \cos x$

$$2 \sin^2 x - 5 \cos x = \cos^2 x$$

$$2 - 2 \cos^2 x - 5 \cos x = \cos^2 x$$

$$3 \cos^2 x + 5 \cos x - 2 = 0$$

- M1** Use $\tan x \equiv \frac{\sin x}{\cos x}$

- M1** Use $\sin^2 x \equiv 1 - \cos^2 x$

- A1** Show given equation convincingly

3

(ii) $(3 \cos x - 1)(\cos x + 2) = 0$

$$\cos x = \frac{1}{3}$$

$$x = 1.23 \text{ rad}$$

$$x = 5.05 \text{ rad}$$

- M1** Attempt to solve quadratic in $\cos x$
- M1** Attempt to find x from root(s) of quadratic
- A1** Obtain 1.23 rad or 70.5°
- A1** Obtain 5.05 rad or 289° (or $2\pi / 360^\circ$ - their solution)
- SR: B1 B1** for answer(s) only

4

(b) $0.5 \times 0.25x \{ \cos 0 + 2(\cos 0.25 + \cos 0.5 + \cos 0.75) + \cos 1 \}$

- M1** Attempt y -coords for at least 4 of the correct 5 x -coords

- M1** Use correct trapezium rule, any h , for their y values to find area between $x = 0$ and $x = 1$

- M1** Correct h (soi) for their y values

- A1** Obtain 0.837

4

$$\approx 0.837$$

10 (i) $u_{15} = 2 + 14 \times 0.5$ $= 9 \text{ km}$	M1 Attempt use of $a + (n - 1)d$ A1 Obtain 9 km 2
(ii) $u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$	B1 State, or imply, $r = 1.1$ M1 Attempt u_{20} , using ar^{n-1} A1 Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
OR	B1 State, or imply, $r = 1.1$ M1 Attempt to solve $ar^{n-1} = 12$ A1 Obtain $n = 20$ (allow $n \geq 20$) 3
(iii) $\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	B1 State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$ M1 Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve A1 Obtain 26, or 25.2 or better A1 Conclude $n = 26$ only, or equiv eg Day 26 4
(iv) swum = $2 \times 30 = 60 \text{ km}$ run = $\frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$ $= 277.5 \text{ km}$ cycle = $\frac{2(1.1^{30} - 1)}{(1.1 - 1)}$ $= 329.0 \text{ km}$ total = 666 km	B1 Obtain 60 km, or $2 \times 30 \text{ km}$ M1 Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$ M1 Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$ A1 Obtain 666 or 667 km 4

4723 Core Mathematics 3

<p>1 <u>Either</u>: Obtain $x = 0$ Form linear equation with signs of $4x$ and $3x$ different State $4x - 5 = -3x + 5$ Obtain $\frac{10}{7}$ and no other non-zero value(s)</p>	<p>B1 ignoring errors in working M1 ignoring other sign errors A1 or equiv without brackets A1 or exact equiv</p>												
<p><u>Or</u>: Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$ Attempt solution of quadratic equation Obtain $\frac{10}{7}$ and no other non-zero value(s) Obtain 0</p>	<p>4 B1 or equiv M1 at least as far as factorisation or use of formula A1 or exact equiv B1 ignoring errors in working 4</p>												
<p>2 (i) Show graph indicating attempt at reflection in $y = x$ Show correct graph with x-coord 2 and y-coord -3 indicated</p>	<p>M1 with correct curvature and crossing negative y-axis and positive x-axis A1 2</p>												
<p>(ii) Show graph indicating attempt at reflection in x-axis Show correct graph with x-coord -3 indicated ... and y-coord -4 indicated [SC: Incorrect curve earning M0 but both correct intercepts indicated</p>	<p>M1 with correct curvature and crossing each negative axis A1 A1 B1] 3</p>												
<p>3 Attempt use of product rule Obtain $2x \ln x + x^2 \cdot \frac{1}{x}$ Substitute e to obtain $3e$ for gradient Attempt eqn of straight line with numerical gradient Obtain $y - e^2 = 3e(x - e)$ Obtain $y = 3ex - 2e^2$</p>	<p>M1 ... + ... form A1 or equiv A1 or exact (unsimplified) equiv M1 allowing approx values A1√ or equiv; following their gradient provided obtained by diffn attempt; allow approx values A1 in terms of e now and in requested form 6</p>												
<p>4 (i) Differentiate to obtain form $kx(2x^2 + 9)^n$ Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$ Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$</p>	<p>M1 any constant k; any $n < \frac{5}{2}$ A1 or (unsimplified) equiv A1 AG; necessary detail required 3</p>												
<p>(ii) Attempt relevant calculations with 0.3 and 0.4 Obtain at least one correct value Obtain two correct values and conclude appropriately</p>	<p>M1 A1</p> <table border="1" data-bbox="973 1702 1356 1814"> <thead> <tr> <th>x</th> <th>$f(x)$</th> <th>$x - f(x)$</th> <th>$f'(x)$</th> </tr> </thead> <tbody> <tr> <td>0.3</td> <td>0.3595</td> <td>-0.0595</td> <td>83.4</td> </tr> <tr> <td>0.4</td> <td>0.3515</td> <td>0.0485</td> <td>113.8</td> </tr> </tbody> </table> <p>A1 noting sign change or showing $0.3 < f(0.3)$ and $0.4 > f(0.4)$ or showing gradients either side of 100 3</p>	x	$f(x)$	$x - f(x)$	$f'(x)$	0.3	0.3595	-0.0595	83.4	0.4	0.3515	0.0485	113.8
x	$f(x)$	$x - f(x)$	$f'(x)$										
0.3	0.3595	-0.0595	83.4										
0.4	0.3515	0.0485	113.8										

(iii) Obtain correct first iterate Carry out correct process Obtain 0.3553	B1 M1 finding at least 3 iterates in all A1 answer required to exactly 4 dp
$[0.3 \rightarrow 0.35953 \rightarrow 0.35497 \rightarrow 0.35534 \rightarrow 0.35531;$ $0.35 \rightarrow 0.35575 \rightarrow 0.35528 \rightarrow 0.35532 (\rightarrow 0.35531);$ $0.4 \rightarrow 0.35146 \rightarrow 0.35563 \rightarrow 0.35529 \rightarrow 0.35532]$	
5 (a) Obtain expression of form $\frac{a \tan \alpha}{b + c \tan^2 \alpha}$	M1 any non-zero constants a, b, c
State correct $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$	A1 or equiv
Attempt to produce polynomial equation in $\tan \alpha$	M1 using sound process
Obtain at least one correct value of $\tan \alpha$	A1 $\tan \alpha = \pm \sqrt{\frac{4}{5}}$
Obtain 41.8	A1 allow 42 or greater accuracy; allow 0.73
Obtain 138.2 and no other values between 0 and 180	A1 allow 138 or greater accuracy
[SC: Answers only 41.8 or ... B1; 138.2 or ... and no others B1]	6
(b)(i) State $\frac{7}{6}$	B1
1	
(ii) Attempt use of identity linking $\cot^2 \beta$ and $\operatorname{cosec}^2 \beta$	M1 or equiv retaining exactness; condone sign errors
Obtain $\frac{13}{36}$	A1 or exact equiv
2	
6 Integrate $k_1 e^{nx}$ to obtain $k_2 e^{nx}$	M1 any constants involving π or not; any n
Obtain correct indefinite integral of their $k_1 e^{nx}$	A1
Substitute limits to obtain $\frac{1}{6}\pi(e^3 - 1)$ or $\frac{1}{6}(e^3 - 1)$	A1 or exact equiv perhaps involving e^0
Integrate $k(2x - 1)^n$ to obtain $k'(2x - 1)^{n+1}$	M1 any constants involving π or not; any n
Obtain correct indefinite integral of their $k(2x - 1)^n$	A1
Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	A1 or exact equiv
Apply formula $\int \pi y^2 dx$ at least once	B1 for $y = e^{3x}$ and/or $y = (2x - 1)^4$
y^2 Subtract, correct way round, attempts at volumes	M1 allow with π missing but must involve
Obtain $\frac{1}{6}\pi e^3 - \frac{2}{9}\pi$	A1 or similarly simplified exact equiv
9	
7 (i) State $A = 42$	B1
State $k = \frac{1}{9}$	B1 or 0.11 or greater accuracy
Attempt correct process for finding m	M1 involving logarithms or equiv
Obtain $\frac{1}{9}\ln 2$ or 0.077	A1 or 0.08 or greater accuracy
4	
(ii) Attempt solution for t using either formula	M1 using correct process (log's or T&I or ...)
Obtain 11.3	A1 or greater accuracy; allow 11.3 ± 0.1
2	
(iii) Differentiate to obtain form $B e^{mt}$	M1 where B is different from A
Obtain $3.235e^{0.077t}$	A1 or equiv; following their A and m
Obtain 47.9	A1 allow 48 or greater accuracy
3	

4723

Mark Scheme

June 2015

<p>8 (i) Show at least correct $\cos \theta \cos 60 + \sin \theta \sin 60$ or $\cos \theta \cos 60 - \sin \theta \sin 60$ Attempt expansion of both with exact numerical values attempted Obtain $\frac{1}{2}\sqrt{3} \sin \theta + \frac{5}{2} \cos \theta$</p>	<p>B1 M1 and with $\cos 60 \neq \sin 60$ A1 or exact equiv 3</p>
<p>(ii) Attempt correct process for finding R Attempt recognisable process for finding α Obtain $\sqrt{7} \sin(\theta + 70.9)$</p>	<p>M1 whether exact or approx M1 allowing sin / cos muddles A1 allow 2.65 for R; allow 70.9 ± 0.1 for α 3</p>
<p>(iii) Attempt correct process to find any value of θ + their α Obtain any correct value for $\theta + 70.9$ Attempt correct process to find θ + their α in 3rd quadrant Obtain 131 [SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]</p>	<p>M1 A1 -158, -22, 202, 338, ... M1 or several values including this A1 or greater accuracy and no other 4</p>
<p>9 (i) Attempt use of quotient rule Obtain $\frac{75 - 15x^2}{(x^2 + 5)^2}$ Equate attempt at first derivative to zero and rearrange to solvable form Obtain $x = \sqrt{5}$ or 2.24 Recognise range as values less than y-coord of st pt Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$</p>	<p>*M1 or equiv; allow u / v muddles A1 or (unsimplified) equiv; this M1A1 available at any stage of question M1 dep *M A1 or greater accuracy M1 allowing < here A1 any notation; with \leq now; any exact equiv 6</p>
<p>(ii) State $\sqrt{5}$</p>	<p>B1 following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k 1</p>
<p>(iii) Equate attempt at first derivative to -1 and attempt simplification Obtain $x^4 - 5x^2 + 100 = 0$ Attempt evaluation of discriminant or equiv Obtain -375 or equiv and conclude appropriately</p>	<p>*M1 and dependent on first M in part (i) A1 or equiv involving 3 non-zero terms M1 dep *M A1 4</p>

4724 Core Mathematics 4

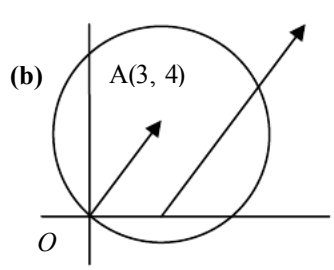
<p>1 (a) $2x^2 - 7x - 4 = (2x+1)(x-4)$ or $3x^2 + x - 2 = (3x-2)(x+1)$</p> <p>$\frac{2x+1}{3x-2}$ as final answer; this answer only</p>	<p>B1</p> <p>B1 Do not ISW</p> <p>2</p>
<p>(b) For correct leading term x in quotient For evidence of correct division process Quotient = $x - 2$</p> <p>Remainder = $x - 3$</p>	<p>B1 <u>Identity method</u></p> <p>M1 M1: $x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R$</p> <p>A1 M1: $Q = ax + b$ or $x + b$, $R = cx + d$ & ≥ 2 ops [N.B. If $Q = x + b$, this \Rightarrow 1 of the 2 ops]</p> <p>A1 A2: $a = 1, b = -2, c = 1, d = -3$ SR: <u>B1</u> for two</p> <p>4</p>
<p>2 Parts with correct split of $u = \ln x$, $\frac{dv}{dx} = x^4$</p> <p>$\frac{x^5}{5} \ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} (dx)$</p> <p>$\frac{x^5}{5} \ln x - \frac{x^5}{25}$</p> <p>Correct method with the limits $\frac{4e^5}{25} + \frac{1}{25}$ ISW (Not '+c')</p>	<p>*M1 obtaining result $f(x) + /- \int g(x) dx$</p> <p>A1</p> <p>A1</p> <p>dep*M1 Decimals acceptable here</p> <p>A1 Accept equiv fract; like terms amalgamated</p> <p>5</p>
<p>3 (i) $\frac{d}{dx}(x^2y) = x^2 \frac{dy}{dx} + 2xy$ or $\frac{d}{dx}(xy^2) = 2xy \frac{dy}{dx} + y^2$</p> <p>Attempt to solve their differentiated equation for $\frac{dy}{dx}$</p> <p>$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ only</p>	<p>*B1</p> <p>dep*M1</p> <p>A1 WWW AG Must have intermediate line &... ...could imply "=0" on 1st line</p> <p>3</p>
<p>(ii)(a) Attempt to solve only $y^2 - 2xy = 0$ & derive $y = 2x$ Clear indication why $y = 0$ is not acceptable</p>	<p>B1 AG Any effort at solving $x^2 - 2xy = 0 \rightarrow B0$</p> <p>B1 Substituting $y = 2x \rightarrow B0, B0$</p> <p>2</p>
<p>(b) Attempt to solve $y = 2x$ simult with $x^2y - xy^2 = 2$ Produce $-2x^3 = 2$ or $y^3 = -8$ $(-1, -2)$ or $x = -1, y = -2$ only</p>	<p>M1</p> <p>A1 AEF</p> <p>A1</p> <p>3</p>

4 (i) For (either point) + t (difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	M1 A1	t can be 's', 'λ' etc. \mathbf{r} must be ' \mathbf{r} ' but need not be bold Check other formats, e.g. $ta + (1-t)b$
2		
(ii) State/imply that their \mathbf{r} and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular Consider scalar product = 0 Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$ Subst their t into their equation of AB Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	*M1 N.B.This *M1 is dep on M1 being earned in (i) dep*M1 A1 M1 A1 Accept decimals if clear	
5		
5 (i) $(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring x^3 etc $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring x^3 etc Product = $1 - x + \frac{1}{2}x^2$ ignoring x^3 etc	B2 SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq -\frac{1}{8}$ or 0 B2 SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq \frac{3}{8}$ or 0 B1 AG; with (at least) 1 intermediate step (cf x^2)	
5		
(ii) $\frac{\sqrt{5}}{9}$ or $\frac{\sqrt{5}}{3}$ seen $\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen $\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	B1 B1 B1 AG	
3		
6 (i) Produce at least 2 of the 3 relevant equations in t and s Solve for t and s $(t, s) = (4, -3)$ AEF Subst $(4, -3)$ into suitable equation(s) & show consistency	M1 $1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$ M1 *A1 dep* A1 Either into "3 rd " eqn or into all 3 coordinates. N.B. Intersection coords not asked for	
4		
(ii) Method for finding magnitude of any vector Method for finding scalar product of any 2 vectors Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a} \mathbf{b} }$ AEF for the correct 2 vectors 137 (136.8359) or 43.2(43.164...)	*M1 Expect $\sqrt{29}$ and $\sqrt{21}$ *M1 Expect -18 dep* M1 Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$ A1 2.39 (2.388236..) or 0.753(0.75335...) rads	
4		

7 (i)	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	M1	
	Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2} \cos x$	A1	
	Show manipulation to $-\operatorname{cosec} x \cot x$ (or vice-versa)	A1	WWW AG with ≥ 1 line intermed working
			3
<hr/>			
(ii)	Separate variables, $\int (-)\frac{1}{\sin x \tan x} dx = \int \cot t dt$	M1	or $\int \frac{1}{\sin x \tan x} dx = \int (-)\cot t dt$
	<u>Style:</u> For the M1 to be awarded, dx and dt must appear on correct sides or there must be \int sign on both sides		
	$\int -\operatorname{cosec} x \cot x dx = \operatorname{cosec} x (+c)$	A1	or $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$
	$\int \cot t dt = \ln \sin t $ or $\ln \sin t (+c)$	B1	or $\int -\cot t dt = -\ln \sin t $ or $-\ln \sin t $
	Subst $(t, x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	M1	and attempt to find 'c'
	$\operatorname{cosec} x = \ln \sin t + 2$ or $\ln \sin t + 2$	A1	WWW ISW; $\operatorname{cosec} \frac{\pi}{6}$ to be changed to 2
			5
<hr/>			
8 (i)	$A(t+1) + B = 2t$	M1	<u>Beware:</u> correct values for A and/or B can be ...
	$A = 2$	A1	... obtained from a wrong identity
	$B = -2$	A1	<u>Alt method:</u> subst suitable values into given... ...expressions
			3
<hr/>			
(ii)	Attempt to connect dx and dt	M1	But not just $dx = dt$. As AG, look carefully.
	$dx = t dt$ s.o.i. AEF	A1	
	$x + \sqrt{2x-1} \rightarrow \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	B1	Any wrong working invalidates
	$\int \frac{2t}{(t+1)^2} dt$	A1	AG WWW The 'dt' must be present
			4
<hr/>			
(iii)	$\int \frac{1}{t+1} dt = \ln(t+1)$	B1	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t+1$
	$\int \frac{1}{(t+1)^2} dt = -\frac{1}{t+1}$	B1	
	Attempt to change limits (expect 1 & 3) and use f(t)	M1	<u>or</u> re-substitute and use 1 and 5 on g(x)
	$\ln 4 - \frac{1}{2}$	A1	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
			4

9 (i)	$A: \theta = \frac{1}{2}\pi$ (accept 90°) $B: \theta = 2\pi$ (accept 360°)	B1 B2	SR If B0 awarded for point B, allow B1 SR for any angle s.t. $\sin \theta = 0$
		3	
(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\frac{dx}{d\theta} = 2 + 2 \cos 2\theta$ $2 + 2 \cos 2\theta = 4 \cos^2 \theta$ with ≥ 1 line intermed work $\frac{dy}{dx} = \frac{4 \cos \theta}{2 + 2 \cos 2\theta}$ s.o.i. $= \sec \theta$	M1 B1 *B1	or $\frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$ Must be used, not just quoted This & previous line are interchangeable dep* A1 WWW AG
		5	
(iii)	Equating $\sec \theta$ to 2 and producing at least one value of θ	M1	degrees or radians
	$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$	A1	‘Exact’ form required
	$(y =) -2\sqrt{3}$	A1	‘Exact’ form required
		3	

4725 Further Pure Mathematics 1

1 (i)	$\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$	B1 Two elements correct B1 All four elements correct 2
<hr/>		
(ii)	EITHER $\frac{1}{3} \begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$ OR	B1 Both diagonals correct B1 Divide by determinant 2 B1 Solve sim. eqns. 1 st column correct B1 2 nd column correct
<hr/>		
2 (i)	5 0.927 or 53.1°	B1 Correct modulus B1 Correct argument, any equivalent form 2
<hr/>		
(ii)(a)	(b) 	B1 Circle centre A (3, 4) B1 Through O, allow if centre is (4, 3) 2 B1 Half line with +ve slope B1 Starting at (3, 0) B1 Parallel to OA, (implied by correct arg shown) 3
<hr/>		
3 (i)	$\frac{r}{(r+1)!}$	M1 Common denominator of (r + 1)! or r!(r + 1)! A1 Obtain given answer correctly 2
<hr/>		
(ii)	$1 - \frac{1}{(n+1)!}$	M1 Express terms as differences using (i) A1 At least 1 st two and last term correct M1 Show pairs cancelling A1 Correct answer a.e.f. 4
<hr/>		
4		B1 Establish result is true, for n = 1 (or 2 or 3) M1 Attempt to multiply A and A ⁿ , or vice versa M1 Correct process for matrix multiplication A1 Obtain 3 ⁿ⁺¹ , 0 and 1 A1 Obtain ½(3 ⁿ⁺¹ - 1) A1 Statement of Induction conclusion, only if 5 marks earned, but may be in body of working 6

5		M1	Express as difference of two series
		M1	Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1	Correct unsimplified answer
		M1	Attempt to factorise
		A1	At least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(3n+2)(n-1)$	A1	Obtain correct answer
			6
6 (i)	$3 - i$	B1	Conjugate stated
			1
(ii)	<i>EITHER</i>	M1	Use sum of roots
		A1	Obtain correct answer
		M1	Use sum of pairs of roots
		A1	Obtain correct answer
		M1	Use product of roots
		A1	Obtain correct answers
	$a = -8, b = 22, c = -20$		6
	<i>OR</i>	M1	Attempt to find a quadratic factor
		A1	Obtain correct factor
		M1	Expand linear and quadratic factors
		A1A1A1	Obtain correct answers
	$a = -8, b = 22, c = -20$		
	<i>OR</i>	M1	Substitute 1 imaginary & the real root into eqn
		M1	Equate real and imaginary parts
		M1	Attempt to solve 3 eqns.
		A1A1A1	Obtain correct answers
	$a = -8, b = 22, c = -20$		
7 (i)		B1	Enlargement (centre O) scale factor 6
			1
(ii)		B1	Reflection
		B1	Mirror line is $y = x$
			2
(iii)		B1	Stretch in y direction
		B1	Scale factor 6, must be a stretch
			2
(iv)		B1	Rotation
		B1	36.9° clockwise or equivalent
			2

8	$\alpha + \beta = -k$ $\alpha\beta = 2k$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{2}(k - 4)$ $\alpha'\beta' = 1$ $x^2 - \frac{1}{2}(k - 4)x + 1 = 0$	B1 State or use correct value B1 State or use correct value M1 Attempt to express sum of new roots in terms of $\alpha + \beta$, $\alpha\beta$ A1 Obtain correct expression A1 Obtain correct answer a.e.f. B1 Correct product of new roots seen B1ft Obtain correct answer, must be an eqn.
		<div style="border: 1px solid black; padding: 2px; display: inline-block;">7</div> Alternative for last 5 marks M1 Obtain expression for $u = \frac{\alpha}{\beta}$ in terms of k and α or k and β A1 Obtain a correct expression A1 rearrange to get α in terms of u M1 Substitute into given equation A1 Obtain correct answer
9 (i)	$x^2 - y^2 = 5 \text{ and } xy = 6$ $\pm(3 + 2i)$	M1 Attempt to equate real and imaginary parts of $(x + iy)^2$ and $5 + 12i$ A1 Obtain both results M1 Eliminate to obtain a quadratic in x^2 or y^2 M1 Solve a 3 term quadratic & obtain x or y A1 Obtain correct answers as complex nos.
(ii)	$5 - 12i$	B1B1 Correct real and imaginary parts
(iii)	$x^2 = 5 \pm 12i$ $x = \pm(3 \pm 2i)$	<div style="border: 1px solid black; padding: 2px; display: inline-block;">2</div> M1 Attempt to solve a quadratic equation A1 Obtain correct answers A1A1 Each pair of correct answers a.e.f.
		<div style="border: 1px solid black; padding: 2px; display: inline-block;">4</div>

10 (i)

M1 Find value of det **AB****A1** Correct value 2 seen**2**

(ii)

M1 Show correct process for adjoint entries**A1** Obtain at least 4 correct entries in adjoint**B1** Divide by their determinant

$$(\mathbf{AB})^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6-3a & a-6 \end{pmatrix}$$

A1 Obtain completely correct answer**4**

(iii) EITHER

M1 State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ **A1** Obtain $\mathbf{B}^{-1} = (\mathbf{AB})^{-1} \times \mathbf{A}$ **M1** Correct multiplication process seen**A1** Obtain three correct elements



$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$$

A1 All elements correct**5**

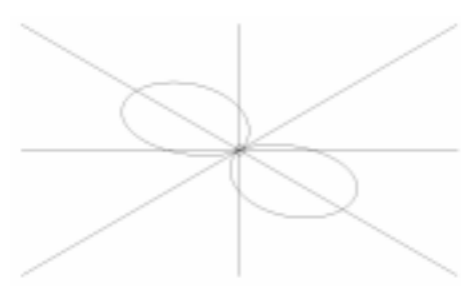
OR

M1 Attempt to find elements of **B****A1** All correct**M1** Correct process for \mathbf{B}^{-1} **A1** 3 elements correct**A1** All elements correct

4726 Further Pure Mathematics 2

<p>1 Write as $\frac{A}{x-2a} + \frac{Bx+C}{x^2+a^2}$ Get $2ax = A(x^2+a^2) + (Bx+C)(x-2a)$ Choose values of x and/or equate coeff. Get $A = \frac{4}{5}, B = -\frac{4}{5}, C = \frac{2}{5}a$</p>	<p>M1 Accept $C=0$ A1√ Follow-on for $C=0$ M1 Must lead to at least one of their A, B, C A1 For two correct from correct working only A1 For third correct 5</p>
<p>2</p> 	<p>B1 Get (4,0), (3,0), (-2,0) only B1 Get (0,√5) as “maximum”</p> <hr/> <p>B1 Meets x-axis at 90° at all crossing points B1 Use $-2 \leq x \leq 3$ and $x \geq 4$ only B1 Symmetry in Ox 5</p>
<p>3 Quote/derive $dx = \frac{2}{1+t^2} dt$ Replace all x and dx from their expressions Tidy to $2/(3t^2+1)$ Get $k \tan^{-1}(At)$ Get $k = \frac{2}{3}\sqrt{3}, A = \sqrt{3}$ Use limits correctly to $\frac{2}{9}, \sqrt{3}\pi$</p>	<p>B1</p> <p>M1 Not $dx=dt$; ignore limits A1 Not $a/(3t^2+1)$ M1 Allow $A=1$ if from $p/(t^2+1)$ only A1√ Allow $k=a/\sqrt{3}$ from line 3; AEEF A1 AEEF 6</p>
<p>4 (i)</p> 	<p>B1 Correct $y = x^2$</p> <hr/> <p>B1 Correct shape/asymptote B1 Crossing (0,1) 3</p>
<p>(ii) Define $\operatorname{sech} x = 2/(e^x + e^{-x})$ Equate their expression to x^2 and attempt to simplify Clearly get A.G.</p>	<p>B1 AEEF M1 A1 3</p>
<p>(iii) Cobweb Values $>$ and then $<$ root</p>	<p>B1 B1 Only from cobweb 2</p>

5 (i) Factorise to $\tan^{n-2}x(1+\tan^2x)$ Clearly use $1+\tan^2 = \sec^2$ Integrate to $\tan^{n-1}x/(n-1)$ Use limits and tidy to A.G.	B1 Or use $\tan^n x = \tan^{n-2}x \cdot \tan^2 x$ M1 Allow wrong sign A1 Quote or via substitution A1 Must be clearly derived 4
(ii) Get $3(I_4 + I_2) = 1, I_2 + I_0 = 1$ Attempt to evaluate I_0 (or I_2) Get $\frac{1}{4}\pi$ (or $1 - \frac{1}{4}\pi$) Replace to $\frac{1}{4}\pi - \frac{2}{3}$	B1 Write down one correct from reduction formula M1 $I_2 = a \tan x + b, a, b \neq 0$ A1 A1 4
6 (i) Attempt to use N-R of correct form with clear $f'(x)$ used Get 2.633929, 2.645672	M1 A1 For one correct to minimum of 6 d.p. A1 √ For other correct from their x_2 in correct NR 3
(ii) $\sqrt{7}$	B1 Allow \pm 1
(iii) Get $e_1 = 0.14575, e_2 = 0.01182$ Get $e_3 = 0.00008$ Verify both ≈ 0.00008	B1 √ From their values B1 √ B1 From 0.000077.. or $0.01182^3/0.14575^2$ 3
7 (i) Attempt quotient/product on bracket Get $-3/(2+x)^2$ Use Formulae Booklet or derive from $\tanh y = (1-x)/(2+x)$ Get $\frac{-3}{(2+x)^2} \cdot \frac{1}{1 - ((1-x)/(2+x))^2}$ Clearly tidy to A.G. Get $f''(x) = 2/(1+2x)^2$	M1 A1 May be implied M1 Attempt \tanh^{-1} part in terms of x A1 √ From their results above A1 B1 cao 6
	SC Use reasonable \ln definition M1 Get $y = \frac{1}{2}\ln((1-k)/(1+k))$ for $k = (1-x)/(1+2x)$ A1 Tidy to $y = \frac{1}{2}\ln(3/(1+2x))$ A1 Attempt chain rule M1 Clearly tidy to A.G. A1 Get $f''(x)$ B1
(ii) Attempt $f(0), f'(0)$ and $f''(0)$ Get $\tanh^{-1} \frac{1}{2}, -1$ and 2 Replace $\tanh^{-1} \frac{1}{2} = \frac{1}{2} \ln 3 (= \ln \sqrt{3})$ Get $\ln \sqrt{3} - x + x^2$	M1 From their differentiation A1 √ B1 Only A1 4 SC Use standard expansion from $\frac{1}{2}\ln 3 - \frac{1}{2}\ln(1+2x)$

8 (i) Attempt to solve $r = 0$ Get $\alpha = \frac{1}{4}\pi$	M1 A1 From correct method; ignore others; allow θ 2
(ii) (a) Get $1 - \sin((2k+1)\pi - 2\theta)$ Expand as $\sin(A+B)$ Use k as integer so $\sin(2k+1)\pi = 0$, And $\cos(2k+1)\pi = -1$	M1 Attempt $f(\frac{1}{2}(2k+1)\pi - \theta)$, leading to 2θ here M1 Or discuss periodicity for general k A1 Needs a clear explanation 3
(b) Quote $\frac{1}{4}(2k+1)\pi$ Select or give $k = 0, 1, 2, 3$	B1 For general answer or 2 correct (ignore other answers given) B1 For all 4 correct in $0 \leq \theta < 2\pi$ 2
(iii) roughly 	B1 Correct shape; 2 branches only, as shown B1 Clear symmetry in correct rays B1 Get max. $r = 2$ B1 At $\theta = \frac{3}{4}\pi$ and $\frac{7}{4}\pi$; both required (allow correct answers not in $0 \leq \theta < 2\pi$ here) 4
9 (i) Attempt to use parts Divide out $x/(1+x)$ Correct answer $x \ln(1+x) - x + \ln(1+x)$ Limits to correct A.G.	M1 Two terms, one yet to be integrated M1 Or use substitution A1 A1 4 SC Quote $\int \ln x \, dx$ M1 Clear use of limits to A.G. A1 SC Attempt to differentiate by product rule M1 Clear use of limits to A.G. A1
(ii) (a) Use sum of areas of rect. Area under curve (between limits 0 and 70) Areas = $1 \times$ heights = $1(\ln 2 + \ln 3 + \dots + \ln 70)$	B1 B1 Areas to be specified 2
(b) Explain use of 69 Explain first rectangle Areas as above $>$ area under curve	B1 Allow diagram or use of left shift of 1 unit B1 B1 3
(c) Show/quote $\ln 2 + \ln 3 + \dots + \ln 70 = \ln 70!$ Use $N = 69, 70$ in (i) Get 228.3, 232.7	B1 M1 No other numbers; may be implied by 228.39.. or 232.65.. seen; allow 228.4, 232.6 or 232.7 A1 3

4727 Further Pure Mathematics 3

1 (a)(i)	e, r^3, r^6, r^9	M1	For stating e, r^m (any $m \geq 2$), and 2 other different elements in terms of e and r
		A1	2 For all elements correct
(ii)	r generates G	B1	1 For this or any statement equivalent to: all elements of G are included in a group with e and r OR order of $r >$ order of all possible proper subgroups
(b)	m, n, p, mn, np, pm	B1	For any 3 orders correct
		B1	2 For all 6 correct and no extras (Ignore 1 and mnp)
5			
2	METHOD 1		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$ ($= 17$)	A1	For correct vector OR LHS of equation
	$\theta = \sin^{-1} \frac{ [1, 4, -1] \cdot [-7, 3, -1] }{\sqrt{1^2 + 4^2 + 1^2} \sqrt{7^2 + 3^2 + 1^2}}$	M1√	For using correct vectors for line and plane f.t. from normal
		M1*	For using scalar product of line and plane vectors
		M1	For calculating both moduli in denominator
	$\theta = \sin^{-1} \frac{6}{\sqrt{18}\sqrt{59}} = 10.6^\circ$	A1√	For scalar product. f.t. from their numerator
	(10.609...°, 0.18517...)	(*dep)	
		A1	7 For correct angle
7			
	METHOD 2		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$	A1	For correct vector OR LHS of equation
	$7x - 3y + z = 17$	M1√	For attempting to find RHS of equation f.t. from \mathbf{n} or LHS of equation
	$d = \frac{ 21 - 12 + 2 - 17 }{\sqrt{7^2 + 3^2 + 1^2}} = \frac{6}{\sqrt{59}}$	M1	For using distance formula from a point on the line, e.g. (3, 4, 2), to the plane
		A1√	For correct distance. f.t. from equation
	$\theta = \sin^{-1} \frac{\frac{6}{\sqrt{59}}}{\sqrt{1^2 + 4^2 + 1^2}} = 10.6^\circ$	M1	For using trigonometry
	(10.609...°, 0.18517...)	A1	For correct angle
7			
3 (i)	$\frac{dz}{dx} = 1 + \frac{dy}{dx}$	M1	For differentiating substitution (seen or implied)
	$\frac{dz}{dx} - 1 = \frac{z+3}{z-1} \Rightarrow \frac{dz}{dx} = \frac{2z+2}{z-1} = \frac{2(z+1)}{z-1}$	A1	For correct equation in z AEF
		A1	3 For correct simplification to AG
(ii)	$\int \frac{z-1}{z+1} dz = 2 \int dx$	B1	For $\int \frac{z-1}{z+1} (dz)$ and $\int (1) (dx)$ seen or implied
	$\Rightarrow \int 1 - \frac{2}{z+1} dz$ OR $\int 1 - \frac{2}{u} du = 2x (+c)$	M1	For rearrangement of LHS into integrable form OR substitution e.g. $u = z+1$ or $u = z-1$
	$\Rightarrow z - 2 \ln(z+1)$ OR $z+1 - 2 \ln(z+1)$	A1	For correct integration of LHS as $f(z)$
	$= 2x (+c)$		
	$\Rightarrow -2 \ln(x+y+1) = x - y + c$	A1	4 For correct general solution AEF

7

4 (i)	$\cos^5 \theta = \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right)^5$	B1	For $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ seen or implied z may be used for $e^{i\theta}$ throughout
	$\cos^5 \theta = \frac{1}{32} (e^{i\theta} + e^{-i\theta})^5$	M1	For expanding $(e^{i\theta} + e^{-i\theta})^5$. At least 3 terms and 2 binomial coefficients required <i>OR</i> reasonable attempt at expansion in stages
	$\cos^5 \theta = \frac{1}{32} (e^{5i\theta} + e^{-5i\theta} + 5(e^{3i\theta} + e^{-3i\theta}) + 10(e^{i\theta} + e^{-i\theta}))$	A1	For correct binomial expansion
	$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$	M1 A1	For grouping terms and using multiple angles For answer obtained correctly AG
<hr style="border-top: 1px dashed black;"/>			
(ii)	$\cos \theta = 16 \cos^5 \theta$	B1	For stating correct equation of degree 5 <i>OR</i> $1 = 16 \cos^4 \theta$ AEF
	$\Rightarrow \cos \theta = 0, \quad \cos \theta = \pm \frac{1}{2}$	M1	For obtaining at least one of the values of $\cos \theta$ from $\cos \theta = k \cos^5 \theta$ <i>OR</i> from $1 = k \cos^4 \theta$
	$\Rightarrow \theta = \frac{1}{2} \pi, \frac{1}{3} \pi, \frac{2}{3} \pi$	A1 A1	A1 for any two correct values of θ A1 4 A1 for the 3rd value and no more in $0, \theta, \pi$ Ignore values outside $0, \theta, \pi$

5 (i) METHOD 1

Lines meet where

$$(x =) k + 2\lambda = k + \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

$$\Rightarrow \lambda = -1, \mu = -2$$

M1 For attempting to solve any 2 equations
A1 For correct values of λ and μ

$$\Rightarrow (k - 2, 4, 4)$$

B1 For attempting a check in 3rd equation
OR verifying point of intersection is on both lines
A1 6 For correct point of intersection (allow vector)
SR For finding λ OR μ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1

METHOD 2

$$d = \frac{|[0, 3, 1] \cdot [2, -5, -3] \times [1, -4, -2]|}{|\mathbf{b} \times \mathbf{c}|}$$

For using $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ with appropriate vectors (division by $|\mathbf{b} \times \mathbf{c}|$ is not essential)

$$d = c[0, 3, 1] \cdot [-2, 1, -3] = 0$$

B1 and showing $d = 0$ correctly \Rightarrow lines intersect

Lines meet where

$$(x =) (k +) 2\lambda = (k +) \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

$$\Rightarrow \lambda = -1, \mu = -2$$

M1 For attempting to solve any 2 equations
A1 For correct value of λ OR μ

$$\Rightarrow (k - 2, 4, 4)$$

A1 For correct point of intersection (allow vector)

METHOD 3

$$\text{e.g. } x - k = \frac{2(y+1)}{-5} = \frac{y+4}{-4}$$

M1 For solving one pair of simultaneous equations

$$\Rightarrow y = 4$$

A1 For correct value of x, y or z

$$\frac{z-1}{-3} = \frac{y+1}{-5}$$

M1 For solving for the third variable

$$x = k - 2 \text{ OR } z = 4$$

A1 For correct values of 2 of x, y and z

$$x - k = \frac{z}{-2} \text{ checks with } x = k - 2, z = 4$$

B1 For attempting a check in 3rd equation

$$\Rightarrow (k - 2, 4, 4)$$

A1 For correct point of intersection (allow vector)

(ii) METHOD 1

$$\mathbf{n} = [2, -5, -3] \times [1, -4, -2]$$

M1 For finding vector product of 2 directions

$$\mathbf{n} = c[-2, 1, -3]$$

A1 For correct normal
SR Following Method 2 for (i), award M1 A1√ for \mathbf{n} , f.t. from their \mathbf{n}

$$(1, -1, 1) \text{ OR } (1, -4, 0) \text{ OR } (-1, 4, 4)$$

M1 For substituting a point in LHS

$$\Rightarrow 2x - y + 3z = 6$$

A1 4 For correct equation of plane **AEF cartesian**

METHOD 2

$$\mathbf{r} = [1, -1, 1] + \lambda[2, -5, -3] + \mu[1, -4, -2]$$

M1 For using vector equation of plane (OR $[1, -4, 0]$ for **a**)

$$x = 1 + 2\lambda + \mu$$

$$y = -1 - 5\lambda - 4\mu$$

A1 For writing 3 linear equations

$$z = 1 - 3\lambda - 2\mu$$

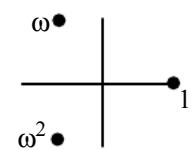

M1 For eliminating λ and μ

$$\Rightarrow 2x - y + 3z = 6$$

A1 For correct equation of plane **AEF cartesian**

6 (i)	When a, b have opposite signs, $a b = \pm ab, b a = \mp ba \Rightarrow a b \neq b a $	M1 A1 2	For considering sign of $a b $ OR $b a $ in general or in a specific case For showing that $a b \neq b a $ Note that $ x = \sqrt{x^2}$ may be used
(ii)	$(a \circ b) \circ c = (a b) \circ c = a b c $ OR $a bc $ $a \circ (b \circ c) = a \circ (b c) = a b c = a b c $ OR $a bc $	M1 A1 M1 A1 4	For using 3 distinct elements and simplifying $(a \circ b) \circ c$ OR $a \circ (b \circ c)$ For obtaining correct answer For simplifying the other bracketed expression For obtaining the same answer
(iii)	<i>EITHER</i> $a \circ e = a e = a \Rightarrow e = \pm 1$ <i>OR</i> $e \circ a = e a = a$ $\Rightarrow e = 1$ for $a > 0, e = -1$ for $a < 0$ Not a group	B1* M1 A1 B1 (*dep) 4	For stating $e = \pm 1$ OR no identity For attempting algebraic justification of +1 and -1 for e For deducing no (unique) identity For stating not a group

10

7 (i)		B1 1	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">Polar or cartesian values of ω and ω^2 may be used anywhere in this question</div> For showing 3 points in approximately correct positions Allow ω and ω^2 interchanged, or unlabelled
(ii)	<i>EITHER</i> $1 + \omega + \omega^2$ = sum of roots of cubic = 0 <i>OR</i> $\omega^3 = 1 \Rightarrow (\omega - 1)(\omega^2 + \omega + 1) = 0$ $\Rightarrow 1 + \omega + \omega^2 = 0$ (for $\omega \neq 1$) <i>OR</i> sum of G.P. $1 + \omega + \omega^2 = \frac{1 - \omega^3}{1 - \omega} \left(= \frac{0}{1 - \omega} \right) = 0$	M1 A1 2	For result shown by any correct method AG
	<i>OR</i>  shown on Argand diagram or explained in terms of vectors		Reference to vectors in part (i) diagram may be made
	<i>OR</i> $1 + \text{cis } \frac{2\pi}{3} + \text{cis } \frac{4\pi}{3} = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$		
(iii) (a)	$(2 + \omega)(2 + \omega^2) = 4 + 2(\omega + \omega^2) + \omega^3$ $= 4 - 2 + 1 = 3$	M1 A1 2	For using $1 + \omega + \omega^2 = 0$ <i>OR</i> values of ω , ω^2 For correct answer
(b)	$\frac{1}{2 + \omega} + \frac{1}{2 + \omega^2} = \frac{2 + (\omega + \omega^2) + 2}{3} = 1$	M1 A1√ 2	For combining fractions <i>OR</i> multiplying top and bottom of 2 fractions by complex conjugates For correct answer f.t. from (a)
(iv)	For the cubic $x^3 + px^2 + qx + r = 0$		
	METHOD 1		
	$\sum \alpha = 2 + 1 = 3$ ($\Rightarrow p = -3$)	M1	For calculating two of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$
	$\sum \alpha\beta = \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2} + \frac{1}{3} = \frac{7}{3}$ ($= q$)	M1	For calculating all of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ <i>OR</i> all of p, q, r
	$\alpha\beta\gamma = \frac{2}{3}$ ($\Rightarrow r = -\frac{2}{3}$)	A1	For at least two of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ correct (or values of p, q, r)
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1 4	For correct equation CAO
	METHOD 2		
	$\left(x - 2\right)\left(x - \frac{1}{2 + \omega}\right)\left(x - \frac{1}{2 + \omega^2}\right) = 0$		
	$x^3 + \left(-2 - \frac{1}{2 + \omega} - \frac{1}{2 + \omega^2}\right)x^2$	M1	For multiplying out LHS in terms of ω or $\text{cis } \frac{1}{3}k\pi$
	$+ \left(\frac{1}{(2 + \omega)(2 + \omega^2)} + \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2}\right)x$		
	$-\frac{2}{(2 + \omega)(2 + \omega^2)} = 0$	M1	For simplifying, using parts (ii), (iii) or values of ω
	$\Rightarrow x^3 - 3x^2 + \frac{7}{3}x - \frac{2}{3} = 0$	A1	For at least two of p, q, r correct
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1	For correct equation CAO

8 (i)	$m^2 + 1 = 0 \Rightarrow m = \pm i$ \Rightarrow C.F. $(y =) Ce^{ix} + De^{-ix} = A \cos x + B \sin x$	M1 A1 2	For stating and attempting to solve correct auxiliary equation For correct C.F. (must be in trig form) SR If some or all of the working is omitted, award full credit for correct answer
(ii)(a)	$y = p(\ln \sin x) \sin x + qx \cos x$ $\frac{dy}{dx} = p \frac{\cos x}{\sin x} \sin x + p(\ln \sin x) \cos x + q \cos x - qx \sin x$ $\frac{d^2y}{dx^2} = -p \sin x - p(\ln \sin x) \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x - qx \cos x$	M1 A1	For attempting to differentiate P.I. (product rule needed at least once) For correct (unsimplified) result AEF
	$-p \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x \equiv \frac{1}{\sin x}$	M1	For substituting their $\frac{d^2y}{dx^2}$ and y into D.E.
	$\Rightarrow p - 2(p+q) \sin^2 x \equiv 1$	M1	For using $\sin^2 x + \cos^2 x = 1$
(b)		A1 6	For simplifying to AG (\equiv may be =)
	$p = 1, q = -1$	M1	For attempting to find p and q by equating coefficients of constant and $\sin^2 x$ AND/OR giving value(s) to x (allow any value for x , including 0)
(iii)	G.S. $y = A \cos x + B \sin x + (\ln \sin x) \sin x - x \cos x$	A1 2	For both values correct
	$\operatorname{cosec} x$ undefined at $x = 0, \pi, 2\pi$ $\text{OR } \sin x > 0$ in $\ln \sin x$ $\Rightarrow 0 < x < \pi$	B1√ M1 A1 3	For correct G.S. f.t. from their C.F. and P.I. with 2 arbitrary constants in C.F. (allow given form of P.I. if p and q have not been found) For considering domain of $\operatorname{cosec} x$ OR $\sin x \neq 0$ OR $\ln \sin x$ term For stating correct range CAO SR Award B1 for correct answer with justification omitted or incorrect

4728 Mechanics 1

1(i)	$900a = 600 - 240$ $a = 0.4 \text{ ms}^{-2}$	AG	M1 A1 [2]	N2L with difference of 2 forces, accept 360
(ii)	$9 = 5 + 0.4t$ $t = 10 \text{ s}$ $9^2 = 5^2 + 2 \times 0.4s$ $s = 70 \text{ m}$		M1 A1 M1 A1 [4]	$v = u + 0.4t$ or $v = u + (cv \ 0.4)t$ or $s = (u+v)t/2$ or $s = ut + 0.5 \times cv(0.4)t^2$
2(i)	Resolves a force in 2 perp. directions Uses Pythagoras $R^2 = (14\sin 30)^2 + (12+14\cos 30)^2$ {or $R^2 = (12\sin 30)^2 + (14+12\cos 30)^2$ } $R = 25.1$	AG	M1* D*M1 A1 A1	<i>Uses vector addition or subtraction</i> <i>Uses cosine rule $R^2 = 14^2 + 12^2 - 2 \times 14 \times 12 \cos 150$</i>
(ii)	Trig to find angle in a valid triangle $\tan B = 7/24.1, \sin B = 7/25.1, \cos B = 24.1/25.1$ $B = 016, (0)16.1^\circ$ or $(0)16.2^\circ$	AG	A1 [5] M1 A1 A1 [3]	\cos (Treat $R^2 = 14^2 + 12^2 + 2 \times 14 \times 12 \cos 30$ as correct) Angle should be relevant $\sin B/14 = \sin 150/25.1$. Others possible. Cosine rule may give $(0)16.4$, award A1
3(i)	$a = 6/5$ $a = 1.2 \text{ ms}^{-2}$		M1 A1 [2]	Acceleration is gradient idea, for portion of graph Accept 6/5
(ii)	$s = (6 \times 10/2) \times 2$ {or $(6 \times 5/2) \times 4$ }		M1 M1 A1 [3]	Area under graph idea or a formula used correctly Double {Quadruple} journey
(iii)	$s = 60 \text{ m}$ $v = -6 + 1.2(17-15)$ $v = -3.6 \text{ ms}^{-1}$		M1 A1 A1 [3]	$v = u + at$ idea, t not equal to 17 (except $v = 1.2t - 24$) $0 = v + cv(1.2)(20-17), v^2 - 2.4v - 21.6 = 0$, etc SR $v = 3.6$ neither A1, but give both A1 if final answer given is -3.6
4(i)	$F = 15\sin 50 - 15\sin 30 = 3.99 \text{ N}$ Left		M1 A1 B1 [3]	Difference of 2 horizontal components, both < 15 Not 4 or 4.0 Accept reference to 30 degree string May be given in ii if not attempted in i
(ii)	$R = f(30, 15\cos 50, 15\cos 30)$ $R = 30 - 15\cos 50 - 15\cos 30$ $\mu = 3.99/7.36(78)$ $\mu = 0.541$ or 0.542 or 0.543		M1 A1 M1 A1 M1 A1 [5]	Equating 4 vertical forces/components 30g is acceptable $= 7.36(78..)$, treat 30g as a misread Using $F = \mu R$, with $cv(3.99)$ and $cv(7.36(78..))$ Accept 0.54 from correct work, e.g. 4/7.4
5(i)	$2400 \times 5 - 3600 \times 3$ $2400v + 3600v$ $2400 \times 5 - 3600 \times 3 = 2400v + 3600v$ $v = 0.2 \text{ ms}^{-1}$ B		B1 B1 M1 A1 B1 [5]	Award if g included Award if g included Equating momentums (award if g included) Not given if g included or if negative.
(ii)(a)	$+/-(-2400v + 3600v)$ $2400 \times 5 - 3600 \times 3 = -2400v + 3600v$ $v = 1 \text{ ms}^{-1}$		B1 M1 A1	<i>No marks in (ii) if g included</i> Equating momentums if "after" signs differ Do not accept if - sign "lost"
(b)	$I = 2400 \times (5+/-1)$ or $3600 \times (3+/-1)$ $I = 14400 \text{ kgms}^{-1}$		M1 A1 [5]	Product of either mass and velocity change Accept -14400

<p>6(i)</p> $x = 0.01t^4 - 0.16t^3 + 0.72t^2$ $v = dx/dt$ $v = 0.04t^3 - 0.48t^2 + 1.44t$ $v(2) = 1.28 \text{ ms}^{-1}$	<p>AG</p>	<p>M1 A1 A1 [3]</p>	<p>Uses differentiation, ignore +c or $v = 4(0.01t^3) - 3(0.16t^2) + 2(0.72t)$ Evidence of evaluation needed</p>
<p>(ii)</p> $a = dv/dt$ $a = 0.12t^2 - 0.96t + 1.44$ $t^2 - 8t + 12 = 0$	<p>AG</p>	<p>M1 A1 A1 [3]</p>	<p>Uses differentiation or $a = 3(0.04t^2) - 2(0.48t) + 1.44$ Simplifies $0.12t^2 - 0.96t + 1.44 = 0$, (or verifies the roots of QE make acceleration zero)</p>
<p>(iii)</p> $(t - 2)(t - 6) = 0$ $t = 2$ $t = 6$ $v(6) = 0 \text{ ms}^{-1}$		<p>M1 A1 A1 B1</p>	<p>Solves quadratic (may be done in ii <u>if used to find v(6)</u>) Or <i>Factorises v into 3 linear factors</i> M1 $v = 0.04t(t-6)^2$ A1 Identifies $t=6$ A1 Evidence of evaluation needed</p>
<p>(iv)</p> <p>Away from A</p>		<p>[4] B1 B1 B1 B1</p>	<p>Starts at origin Rises to single max, continues through single min Minimum on t axis, non-linear graph</p>
<p>(v)</p> $AB = 0.01x6^4 - 0.16x6^3 + 0.72x6^2$ $AB = 4.32 \text{ m}$		<p>[4] M1 A1 [2]</p>	<p>Or integration of v(t), with limits 0, 6 or substitution, using cv(6) from iii</p>

<p>7(i)</p> $(R) = 0.2 \times 9.8 \cos 45$ $F = 1 \times R = 1 \times 0.2 \times 9.8 \cos 45 = 1.386 \text{ N}$	<p>AG</p>	<p>M1 A1 [2]</p>	<p>Not $F = 0.2 \times 9.8 \cos 45$ or $0.2 \times 9.8 \sin 45$ unless followed by (eg) $F_r = 1 \times F = 1.386$ when M1A1</p>
<p>(ii)</p> <p>Any 1 application of N2L // to plane with correct mass and number of forces</p> $0.4a = 0.2g \sin 45 + 0.2g \sin 45 - 1.38(592..)$ $a = 3.465 \text{ ms}^{-2}$ $0.2a = 0.2g \sin 45 - T \quad \text{or}$ $0.2a = T + [0.2g \sin 45 - 1.38(592..)]$ $T = 0.693 \text{ N}$	<p>AG</p>	<p>M1 A1 A1 M1 A1 [5]</p>	<p>Must use component of weight</p> <p>Accept with 3.465 (or close) instead of a Accept omission of [term] for M1 Accept 0.69</p>
<p>OR</p> <p>Any 1 application of N2L // to plane with correct mass and number of forces</p> $0.2a = 0.2g \sin 45 - T \quad \text{or}$ $0.2a = T + [0.2g \sin 45 - 1.38(592..)]$ <p>Eliminates a or T</p> $a = 3.465 \text{ ms}^{-2}$ $T = 0.693 \text{ N}$	<p>AG</p>	<p>M1 A1 M1 A1 A1</p>	<p>Must use component of weight Either correct Both correct. Accept omission of [term] for A1 only</p>
<p>(iii)</p> $v^2 = 2 \times 3.465 \times 0.5$ $v = 1.86 \text{ ms}^{-1}$		<p>M1 A1 [2]</p>	<p>Using $v^2 = 0^2 + 2xcv(3.465)s$</p>
<p>(iv)</p> <p>For Q</p> $(0.2)a = (0.2)g \sin 45 - (1)(0.2)g \cos 45$ $a = 0$ $T = (3/1.86) = 1.6(12)$ <p>For P</p> $a = 9.8 \sin 45$ $2.5 = 1.86(14..)t + 0.5 \times (9.8 \sin 45)t^2$ $t = 0.6(223)$ $\text{time difference } 1.612 - 0.622 = 0.99(0) \text{ s}$	<p>[AG]</p>	<p>M1 A1 B1 B1 M1 A1 A1 [7]</p>	<p>Attempting equation to find a for Q Accept from $0.2g \sin 45 - 1.386$ Accept 2 sf</p> <p>$a = 6.93$ Using $2.5 = cv(1.86)t + 0.5cv(6.93)t^2$ [not 9.8 or 3.465] Accept 1sf Accept art 0.99 from correct work</p>

4729 Mechanics 2

1	$200\cos 35^\circ$ $200\cos 35^\circ \times d = 5000$ $d = 30.5 \text{ m}$	B1 M1 A1 3		3
2	$0.03R = \frac{1}{2} \times 0.009(250^2 - 150^2)$ $0.03R$	M1 B1	$150^2 = 250^2 + 2a \times 0.03$ $a = \pm 2 \times 10^6 / 3$ or $\pm 666,667$ (A1)	
	either K.E. $R = 6000 \text{ N}$	B1 A1 4	$F = 0.009a$ (M1) unit errors	4
3 (i)	$D = 12000/20$ $12000/20 = k \times 20 + 600 \times 9.8 \times 0.1$ $k = 0.6$	B1 M1 A1 3	AG	
(ii)	$16000/v = 0.6v + 600 \times 9.8 \times 0.1$ $0.6v^2 + 588v - 16000 = 0$ $v = 26.5 \text{ m s}^{-1}$	M1 M1 A1 3	attempt to solve quad. (3 terms)	
(iii)	$16000/32 - 0.6 \times 32 = 600a$ $a = 0.801 \text{ m s}^{-2}$	M1 A1 A1 3	0.80 or 0.8	9
4 (i)	$0 = 35\sin\theta \times t - 4.9t^2$ $t = 35\sin\theta/4.9$ $50\sin\theta/7$ $R = 35\cos\theta \times t$ aef $R = 35^2 \sin\theta \cdot \cos\theta / 4.9$ $R = 125\sin 2\theta$	M1 A1 B1 M1 A1 5	$R = u^2 \sin 2\theta / g$ only ok if proved or $70\sin\theta / g$ aef their t eliminate t	
(ii)	$110 = 125\sin 2\theta$ $\theta = 30.8^\circ$ or 59.2° $t = 3.66 \text{ s}$ or 6.13 s	M1 A1+1 A1+1 5	AG	10
5 (i)	$3/8 \times 3$ (1.125) $0.53d = 5 \times 0.02 + (10 + 3/8 \times 3) \times 0.5$	B1 M1 A1 A1 4	c.o.m. hemisphere $0.53e = 3 \times 5/8 \times 0.5 + 8 \times 0.02 + 13 \times 0.01$ $0.53f = 3 \times 3/8 \times 0.5 - 5 \times 0.02 - 10 \times 0.01$ AG ($e = 2.316$ $f = 0.684$) distance / angle not a complimentary pair	
(ii)	Attempt to calc a pair relevant to P,G $OP = 0.9$ (pair), $p = 73.3^\circ$ $q = 16.7^\circ$ $r = 76.9^\circ$ (77.2°) , $s = 13.1^\circ$ (12.8°) $AC = 0.86$, $BC = 0.67$, $AD = 10.4$ $BD = 10.2$ $r > p$, $s < q$, $p + s < 90$, $0.67 < 0.86$, $10.2 < 10.4$ it is in equilibrium	M1 A1 M1 A1 4	make relevant comparison $0.7 < 0.9$ ($OG < OP$) $10.7 < 10.9$	8

<p>6 (i)</p> <p>$T\cos 60^\circ = S\cos 60^\circ + 4.9$</p> <p>$T\sin 60^\circ + S\sin 60^\circ = 0.5 \times 3^2/0.4$</p> <p>$(S + 9.8)\sin 60^\circ + S\sin 60^\circ = 45/4$</p> <p>$S = 1.60 \text{ N}$</p> <p>$T = 11.4 \text{ N}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>7</p>	<p>Resolving vertically nb for M1: (must be components – all 4 cases) Res. Horiz. $m\omega^2$ ok if $\omega \neq 3$ If equal tensions $2T=45/4$ M1 only</p>	12
<p>(ii)</p> <p>$T\cos 60^\circ = 4.9$</p> <p>$T = 9.8$</p> <p>$T\sin 60^\circ = 0.5 \times 0.4\omega^2$</p> <p>$\omega = 6.51 \text{ rad s}^{-1}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>5</p>	<p>Resolving vertically (component)</p> <p>Resolving horiz. (component)</p> <p>or 6.5</p>	
<p>7 (i)</p> <p>$u = 3 \text{ m s}^{-1}$</p> <p>$6 = 2x + 3y$</p> <p>$e = (y - x) / 3$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>6</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>7</p>	<p>($e = \frac{2}{3}$) (equus must be consistent)</p> <p>AG</p> <p>or (B1) $\frac{1}{2}mx^2$</p> <p>(B1) $\frac{1}{2}mv^2$</p> <p>(B1) $mx \cdot 9.8x4$</p> <p>$v = \sqrt{2^2 + 2 \times 9.8x4}$</p> <p>or $\cos^{-1}(2/9.08)$</p> <p>12.7° to vertical</p>	13
<p>8 (i)</p> <p>com of Δ 3 cm right of C</p> <p>$(48+27)\bar{x} = 48 \times 4 + 27 \times 11$</p> <p>$\bar{x} = 6.52$</p> <p>com of Δ 2 cm above AD</p> <p>$(48+27)\bar{y} = 48 \times 3 + 27 \times 2$</p> <p>$\bar{y} = 2.64$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>8</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>5</p>	<p>can be implied e.g. $7/\sin 30^\circ \cdot F$</p> <p>7.034 (AG) or $(6.52 - 2.64 \tan 30^\circ)$</p> <p>52.0° (GAH) or (above) $x \cos 30^\circ$</p> <p>$(5.00)x \cos 30^\circ$ (4.33)</p> <p>$14F = 3 \times 9.8 \times 7.034 \times \cos 52.0^\circ$</p>	13
<p>(ii)</p> <p>14F</p> <p>$3g \cos 30^\circ \times 6.52$</p> <p>$3g \sin 30^\circ \times 2.64$</p> <p>$14F = 3g \cos 30^\circ \times 6.52 - 3g \sin 30^\circ \times 2.64$</p> <p>$F = 9.09 \text{ N}$</p>			

4730 Mechanics 3

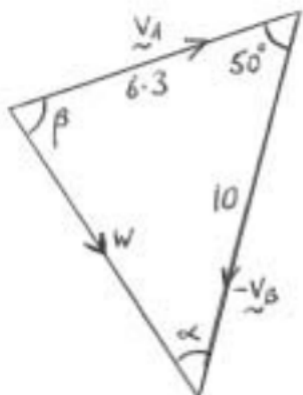
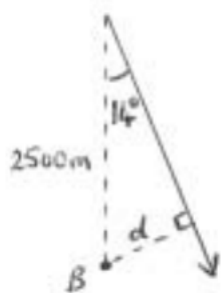
1	(i) $T = (1.35mg)(3 - 1.8) \div 1.8$ [$0.9mg = ma$] Acceleration is $8.82ms^{-2}$	B1 M1 A1	3	For using $T = ma$
	(ii) Initial EE = $(1.35mg)(3 - 1.8)^2 \div (2 \times 1.8)$ [$\frac{1}{2}mv^2 = 0.54mg$] Speed is $3.25ms^{-1}$	B1 M1 A1		
2	(i) Component is $8\sin 27^\circ$ Component is $2.18ms^{-1}$	M1 A1 A1	3	For using NEL vertically
	(ii) Change in velocity vertically = $8\sin 27^\circ(1 + e)$ $ I = 0.2 \times 5.81$ Magnitude of Impulse is 1.16 kgms^{-1}	B1ft M1 A1ft		
3	$0.8 \times 12 \cos 60^\circ = 0.8a + 2b$	M1 A1	10	For using the principle of conservation of momentum in the i direction For using NEL For eliminating b; depends on at least one previous M mark For correct method for speed or direction ft $v^2 = a^2 + 108$ Accept $\theta = 81.8^\circ$ if θ is clearly and appropriately indicated; ft $\tan^{-1} \theta = (12 \sin 60^\circ) / a $
	$0.75 \times 12 \cos 60^\circ = b - a$	M1 A1		
	[$4.8 = 0.8a + 2(a + 4.5)$] $a = -1.5$ Comp. of vel. perp. to l.o.c. after impact is $12 \sin 60^\circ$	DM1 A1 B1		
	The speed of A is $10.5ms^{-1}$	M1 A1ft		
	Direction of A is at 98.2° to l.o.c.	A1ft		

4	(i)	$[mgsin\alpha - 0.2mv = ma]$	M1	For using Newton's second law	
		$5 \frac{dv}{dt} = 28 - v$	A1	AG	
		$[\int \frac{5}{28 - v} dv = \int dt]$	M1	For separating variables and integrating	
		$(C) - 5\ln(28 - v) = t$	A1		
			M1	For using $v = 0$ when $t = 0$	
			A1ft	ft for $\ln[(28 - v)/28] = t/A$ from $C + A\ln(28 - v) = t$ previously	
		$\ln[(28 - v)/28] = -t/5$	A1ft		
		$[28 - v = 28e^{-t/5}]$	M1	For expressing v in terms of t	
		$v = 28(1 - e^{-t/5})$	A1ft	ft for $v = 28(1 - e^{-t/5})$ from $\ln[(28 - v)/28] = t/A$ previously	8
	(ii)			For using $a = (28 - v(t))/5$ or $a = d(28 - 28e^{-t/5})/dt$ and substituting $t = 10$.	
		$[a = 28e^{-2}/5]$	M1	ft from incorrect v in the form $a + be^{ct}$ ($b \neq 0$); Accept $5.6/e^2$	
		Acceleration is $0.758ms^{-2}$	A1ft		2
5	(i)		M1	For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B = 280$ and $F_A = F_B$	
		$1.4R_A = 150 \times 0.95 + 130 \times 0.25$ or			
		$1.4R_B = 130 \times 1.15 + 150 \times 0.45$ or			
		$1.2F - 0.9(280 - R_B) + 0.45 \times 150 - 1.2F +$	A1		
		$0.5R_B$			
		$- 0.25 \times 130 = 0$	A1	AG	
		$R_A = 125N$	A1		
		$R_B = 155N$	B1		4
		(ii)	M1	For taking moments about X for XA or XB	
		$1.2F_A = -150 \times 0.45 + 0.9R_A$ or			
$1.2F_B = 0.5R_B - 130 \times 0.25$	A1				
F_A or $F_B = 37.5N$	A1ft	$F_B = (1.25R_B - 81.25)/3$			
F_B or $F_A = 37.5N$	B1ft		4		
(iii)		Horizontal component is $37.5N$ to the left	B1ft	ft $H = F$ or $H = 56.25 - 0.75V$ or $12H = 325 + 5V$	
		$[Y + R_A = 150]$	M1	For resolving forces on XA vertically	
		Vertical component is $25N$ upwards	A1ft	ft $3V = 225 - 4H$ or $V = 2.4H - 65$	3

6	(i)				
		$[0.36 - 0.144x = 0.1a]$	M1		For applying Newton's second law
		$\ddot{x} = 3.6 - 1.44x$	A1		
		$\ddot{y} = -1.44y \rightarrow \text{SHM}$	or		
		$d^2(x - 2.5) / dt^2 = -1.44(x - 2.5) \rightarrow \text{SHM}$	B1		
			M1		For using $T = 2\pi / n$
		Of period 5.24s	A1	5	AG
	(ii)	Amplitude is 0.5m	B1		
		$0.48^2 = 1.2^2(0.5^2 - y^2)$	M1		For using $v^2 = n^2(a^2 - y^2)$
		Possible values are 2.2 and 2.8	A1ft	4	
	(iii)	$[t_0 = (\sin^{-1}0.6)/1.2; t_1 = (\cos^{-1}0.6)/1.2]$	M1		For using $y = 0.5\sin 1.2t$ to find t_0 or $y = 0.5\cos 1.2t$ to find t_1
		$t_0 = 0.53625 \dots$ or $t_1 = 0.7727 \dots$	A1		Principal value may be implied
	(a)	$[2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1}0.6)/1.2]$	M1		For using $\Delta t = 2t_0$ or $\Delta t = T/2 - 2t_1$
		Time interval is 1.07s	A1ft		ft incorrect t_0 or t_1
	(b)				From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$; ft 2.62 - ans(a) or incorrect t_0 or t_1
		Time interval is 1.55s	B1ft	5	
7	(i)		M1		For using KE gain = PE loss
		$\frac{1}{2}mv^2 = mga(1 - \cos \theta)$	A1		
		$aw^2 = 2g(1 - \cos \theta)$	B1	3	AG From $v = wr$
	(ii)				For using Newton's second law radially (3 terms required) with accel = v^2/r or w^2r
		$mv^2/a = mg\cos \theta - R$ or $maw^2 = mg\cos \theta - R$	M1		
		$[2mg(1 - \cos \theta) = mg\cos \theta - R]$	A1		
		$R = mg(3\cos \theta - 2)$	DM1		For eliminating v^2 or w^2 ; depends on at least one previous M1
			A1ft	4	ft sign error in N2 equation
	(iii)				For using Newton's second law tangentially or differentiating
		$[mg\sin \theta = m(\text{accel.})$ or $2a(\dot{\theta})\ddot{\theta} = 2g\sin \theta(\dot{\theta})]$	M1		$aw^2 = 2g(1 - \cos \theta)$ w.r.t. t
		Accel. ($=a\ddot{\theta}$) = $g\sin \theta$	A1		
		$[\theta = \cos^{-1}(2/3)]$	M1		For using $R = 0$
		Acceleration is 7.30ms^{-2}	A1ft	4	ft from incorrect R of the form $mg(A\cos \square + B)$, $A \neq 0$, $B \neq 0$; accept $g\sqrt{5}/3$
	(iv)				For using rate of change = $(dR/d\theta)(d\theta/dt)$
		$dR/dt = (-3mg\sin \theta) \sqrt{2g(1 - \cos \theta)} / a$	M1		ft from incorrect R of the form $mg(A\cos \square + B)$, $A \neq 0$
			A1ft		For using $\cos \theta = 2/3$
		Rate of change is $-mg \sqrt{\frac{10g}{3a}} \text{Ns}^{-1}$	M1		Any correct form of \dot{R} with $\cos \theta = 2/3$ used; ft with \square from incorrect R of the form $mg(A\cos \square + B)$, $A \neq 0$, $B \neq 0$
			A1ft	4	

4731 Mechanics 4

1	By conservation of angular momentum $1.5 \times 21 + I_G \times 36 = 1.5 \times 28 + I_G \times 34$ $I_G = 5.25 \text{ kg m}^2$	M1 A1A1 A1 4	Give A1 for each side of the equation or $1.5(28 - 21) = I_G(36 - 34)$
2 (i)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$, $0^2 = 8^2 + 2\alpha(2\pi \times 16)$ $\alpha = -\frac{1}{\pi} = -0.318$ Angular deceleration is 0.318 rad s^{-2}	M1 A1 2	Accept $-\frac{1}{\pi}$
(ii)	Using $\omega_1^2 = \omega_0^2 + 2\alpha\theta$, $\omega^2 = 8^2 + 2\alpha(2\pi \times 15)$ $\omega = 2 \text{ rad s}^{-1}$	M1 A1 ft 2	or $0^2 = \omega^2 + 2\alpha(2\pi)$ ft is $\sqrt{64 - 60\pi \alpha }$ or $\sqrt{4\pi \alpha }$ Allow A1 for $\omega = 2$ obtained using $\theta = 16$ and $\theta = 15$ (or $\theta = 1$)
(iii)	Using $\omega_1 = \omega_0 + \alpha t$, $0 = \omega + \alpha t$ $t = 2\pi = 6.28 \text{ s}$	M1 A1 ft 2	or $2\pi = 0t - \frac{1}{2}\alpha t^2$ ft is $\frac{\omega}{ \alpha }$ or $\sqrt{\frac{4\pi}{ \alpha }}$ Accept 2π
3	$A = \int_0^3 (2x + x^2) dx$ $= \left[x^2 + \frac{1}{3}x^3 \right]_0^3 = 18$ $A\bar{x} = \int_0^3 x(2x + x^2) dx$ $= \left[\frac{2}{3}x^3 + \frac{1}{4}x^4 \right]_0^3 = \frac{153}{4} = 38.25$ $\bar{x} = \frac{38.25}{18} = \frac{17}{8} = 2.125$ $A\bar{y} = \int_0^3 \frac{1}{2}(2x + x^2)^2 dx$ $= \int_0^3 (2x^2 + 2x^3 + \frac{1}{2}x^4) dx$ $= \left[\frac{2}{3}x^3 + \frac{1}{2}x^4 + \frac{1}{10}x^5 \right]_0^3 = 82.8$ $\bar{y} = \frac{82.8}{18} = 4.6$	M1 A1 M1 M1 A1 M1 M1 M1 A1 9	Definite integrals may be evaluated by calculator (i.e with no working shown) Integrating and evaluating (dependent on previous M1) or $\int_0^{15} (3 - (\sqrt{y+1} - 1))y dy$ Arranging in integrable form Integrating and evaluating SR If $\frac{1}{2}$ is missing, then M0M1M1A0 can be earned for \bar{y}

<p>4 (i)</p>	 <p> $w^2 = 6.3^2 + 10^2 - 2 \times 6.3 \times 10 \cos 50^\circ$ $w = 7.66 \text{ ms}^{-1}$ $\frac{\sin \alpha}{6.3} = \frac{\sin 50^\circ}{w}$ $\alpha = 39.04^\circ \quad (\beta = 90.96^\circ)$ Bearing is $205 - \alpha = 166^\circ$ </p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>5</p>	<p>Correct velocity triangle</p> <p><i>This mark cannot be earned from work done in part (ii)</i></p>
	<p>OR</p> <p> $\begin{pmatrix} 6.3 \sin 75 \\ 6.3 \cos 75 \end{pmatrix} - \begin{pmatrix} 10 \sin 25 \\ 10 \cos 25 \end{pmatrix} = \begin{pmatrix} 1.859 \\ -7.433 \end{pmatrix}$ </p> <p> $w = \sqrt{1.859^2 + 7.433^2} = 7.66$ Bearing is $180 - \tan^{-1} \frac{1.859}{7.433} = 166^\circ$ </p>	<p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Finding magnitude or direction</p>
<p>(ii)</p>	<p>As viewed from B</p>  <p> $d = 2500 \sin 14.04$ $= 607 \text{ m}$ </p>	<p>B1 ft</p> <p>M1</p> <p>A1</p> <p>3</p>	<p>Diagram showing path of A as viewed from B <i>May be implied</i> Or B1 for a correct (ft) expression for d^2 in terms of t</p> <p>or other complete method Accept 604.8 to 609 SR If $\beta = 89^\circ$ is used, give A1 for 684.9 to 689.1</p>

5 (i)	$V = \int_a^{4a} \pi(ax) dx$ $= \left[\frac{1}{2} \pi a x^2 \right]_a^{4a} = \frac{15}{2} \pi a^3$ <p>Hence $m = \frac{15}{2} \pi a^3 \rho$</p> $I = \sum \frac{1}{2} (\rho \pi y^2 \delta x) y^2 = \int \frac{1}{2} \rho \pi y^4 dx$ $= \int_a^{4a} \frac{1}{2} \rho \pi a^2 x^2 dx$ $= \left[\frac{1}{6} \rho \pi a^2 x^3 \right]_a^{4a} = \frac{21}{2} \rho \pi a^5$ $= \frac{7}{5} \left(\frac{15}{2} \pi a^3 \rho \right) a^2 = \frac{7}{5} m a^2$	M1 M1 M1 M1 A1 A1 ft A1 A1 (ag)	(Omission of π is an accuracy error) For $\int y^4 dx$ Substitute for y^4 and correct limits	8
(ii)	MI about axis, $I_A = \frac{7}{5} m a^2 + m a^2$ $= \frac{12}{5} m a^2$ <p>Period is $2\pi \sqrt{\frac{I}{mgh}}$</p> $= 2\pi \sqrt{\frac{\frac{12}{5} m a^2}{mga}} = 2\pi \sqrt{\frac{12a}{5g}}$	M1 A1 M1 A1 ft	Using parallel axes rule ft from any I with $h = a$	4
6 (i)	$I = \frac{1}{3} m \left\{ a^2 + \left(\frac{3}{2} a \right)^2 \right\} + m \left(\frac{1}{2} a \right)^2$ $= \frac{13}{12} m a^2 + \frac{1}{4} m a^2 = \frac{4}{3} m a^2$	M1 M1 A1 (ag)	MI about perp axis through centre Using parallel axes rule	3
(ii)	By conservation of energy $\frac{1}{2} \left(\frac{4}{3} m a^2 \right) \omega^2 - \frac{1}{2} \left(\frac{4}{3} m a^2 \right) \frac{9g}{10a} = mg \left(\frac{1}{2} a - \frac{1}{2} a \times \frac{3}{5} \right)$ $\frac{2}{3} m a^2 \omega^2 - \frac{3}{5} m g a = \frac{1}{5} m g a$ $\omega^2 = \frac{6g}{5a}$	M1 A1 A1 (ag)	Equation involving KE and PE	3
(iii)	$mg \cos \theta - R = m \left(\frac{1}{2} a \right) \omega^2$ $mg \times \frac{3}{5} - R = \frac{3}{5} mg$ $R = 0$ $mg \left(\frac{1}{2} a \sin \theta \right) = I \alpha$ $\alpha = \frac{3g}{10a}$ $mg \sin \theta - S = m \left(\frac{1}{2} a \right) \alpha$ $S = \frac{4}{5} mg - \frac{3}{20} mg$ $= \frac{13}{20} mg$	M1 A1 A1 (ag) M1A1 A1 M1A1 A1	Acceleration $r\omega^2$ and three terms (one term must be R) SR $mg \cos \theta + R = m \left(\frac{1}{2} a \right) \omega^2 \Rightarrow R = 0$ earns M1A0A1 Applying $L = I\alpha$ Acceleration $r\alpha$ and three terms (one term must be S) or $S \left(\frac{1}{2} a \right) = I_G \alpha = \frac{13}{12} m a^2 \alpha$	9

7 (i)	$U = 3mgx + 2mg(3a - x)$ $+ \frac{mg}{2a}(x - a)^2 + \frac{2mg}{2a}(2a - x)^2$ $= \frac{mg}{2a}(3x^2 - 8ax + 21a^2)$ $\frac{dU}{dx} = 3mg - 2mg + \frac{mg}{a}(x - a) - \frac{2mg}{a}(2a - x)$ $= \frac{3mgx}{a} - 4mg$ <p>When $x = \frac{4}{3}a$, $\frac{dU}{dx} = 4mg - 4mg = 0$ so this is a position of equilibrium</p> $\frac{d^2U}{dx^2} = \frac{3mg}{a}$ > 0 , so equilibrium is stable	B1B1 B1B1 M1 A1 A1 (ag) M1 A1 (ag)	<i>Can be awarded for terms listed separately</i> Obtaining $\frac{dU}{dx}$ <i>(or any multiple of this)</i>
(ii)	<p>KE is $\frac{1}{2}(3m)v^2 + \frac{1}{2}(2m)v^2$</p> <p>Energy equation is $U + \frac{5}{2}mv^2 = \text{constant}$</p> <p>Differentiating with respect to t</p> $\left(\frac{3mgx}{a} - 4mg\right)\frac{dx}{dt} + 5mv\frac{dv}{dt} = 0$ $\frac{3gx}{a} - 4g + 5\frac{d^2x}{dt^2} = 0$ <p>Putting $x = \frac{4}{3}a + y$, $\frac{3gy}{a} + 5\frac{d^2y}{dt^2} = 0$</p> $\frac{d^2y}{dt^2} = -\frac{3g}{5a}y$ <p>Hence motion is SHM with period $2\pi\sqrt{\frac{5a}{3g}}$</p>	M1A1 M1 A1 ft A1 ft M1A1 ft A1 (ag) A1	Differentiating the energy equation (with respect to t or x) <i>Condone \ddot{x} instead of \ddot{y}</i> <i>Award M1 even if KE is missing</i> <i>Must have $\ddot{y} = -\omega^2 y$ or other satisfactory explanation</i>

4732 Probability & Statistics 1

Note: “(3 sfs)” means “answer which rounds to ... to 3 sfs”. If correct ans seen to ≥ 3 sfs, ISW for later rounding
 Penalise over-rounding only once in paper.

1(i)	(a) -1 (b) 0	B1 B1 2	allow ≈ -1 or close to -1 not “strong corr’n”, not -0.99 allow ≈ 0 or close to 0 not “no corr’n”
(ii)	$\begin{matrix} 4 & 3 & 2 & 1 & & \text{or} & 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 & & & 4 & 2 & 1 & 3 \\ \hline \Sigma d^2 & & & & & & (= 14) \\ 1 - \frac{6\Sigma d^2}{4(4^2-1)} \\ = -0.4 \text{ oe} \end{matrix}$	M1 A1 M1 M1 A1 5	Ranks attempted, even if opp Dep M1 or $S_{xy} = 23^{-100/4}$ or $S_{xx} = S_{yy} = 30^{-100/4}$ Dep 2 nd M1 $S_{xy} / \sqrt{(S_{xx}S_{yy})}$
Total		7	
2(i)	$\frac{{}^7C_2 \times {}^8C_3}{{}^{15}C_5}$ $= {}^{56}/_{143}$ or ${}^{1176}/_{3003}$ or 0.392 (3sfs)	M1 M1 A1 3	${}^7C_2 \times {}^8C_3$ or 1176 : M1 $(\text{Any C or P})/{}^{15}C_5$: M1 (dep < 1) or $\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$ or 0.0392: M1 $\times {}^5C_2$ or $\times 10$: M1 (dep ≥ 4 probs mult) if 2 \leftrightarrow 3, treat as MR max M1M1
(ii)	$3! \times 2!$ or ${}^3P_3 \times {}^2P_2$ not in denom $= 12$	M1 A1 2	BABAB seen: M1 120-12: M1A0 NB $4!/2! = 12$: M0A0
Total		5	
3(i)(a)	0.9368 or 0.937	B1 1	
(b)	$0.7799 - 0.5230$ or ${}^8C_5 \times 0.45^3 \times 0.55^5$ $= 0.2569$ or 0.2568 or 0.257	M1 A1 2	Allow 0.9368 – 0.7799
(c)	0.7799 seen – 0.0885 (not 1 – 0.0885) $= 0.691$ (3 sfs)	M1 M1 A1 3	${}^8C_5 \times 0.45^3 \times 0.55^5 + {}^8C_4 \times 0.45^4 \times 0.55^4 + {}^8C_3 \times 0.45^5 \times 0.55^3$: M2 1 term omitted or wrong or extra: M1
(ii)(a)	${}^{10}C_2 \times ({}^7/_{12})^8 \times ({}^7/_{12})^2$ seen $= 0.105$ (3 sfs)	M1 A1 2	or 0.105 seen, but not ISW for A1
(b)	$2^{31/72}$ or ${}^{175}/_{72}$ or 2.43 (3 sfs)	B1 1	NB ${}^{12}/_5 = 2.4$: B0
Total		9	
4(i)	${}^{1/20} \times {}^{1/10}$ or ${}^{1/200}$ or 0.005 $\times 2$ $= {}^{1/100}$ or 0.01	M1 M1dep A1 3	
(ii)	$E(X) = 0 + 50 \times {}^{1/10} + 500 \times {}^{1/20}$ or $0 + 0.5 \times {}^{1/10} + 5 \times {}^{1/20}$ $= 30p$ = £0.30 or ${}^3/_{10}$ Charge “30p” + 20p or 0.3 + 0.2 $= 50p$ or 0.50 or 0.5	M1 A1 M1 A1 4	or eg 20 goes: $2 \times \text{£}0.50 + \text{£}5.00$ $= \text{£}6.00$ $(\text{“£}6.00\text{”} + 20 \times \text{£}0.20) \div 20$ condone muddled units eg 0.3 + 20 $x = 20, 70, 520$: M1A1 $20 \times {}^{17}/_{20} + 70 \times {}^1/_{10} + 520 \times {}^1/_{20}$: M1 $= 50$: A1 $x, (x - 50), (x - 500)$: M1A1 $x \times {}^{17}/_{20} + (x - 50) \times {}^1/_{10} + (x - 500) \times {}^1/_{20} = 20$: M1 $x = 50$: A1 Ignore “£” or “p”
Total		7	

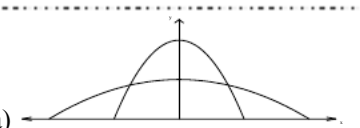
5(i)	$\frac{12}{22} \times \frac{11}{21}$ $= \frac{2}{7}$ oe or 0.286 (3 sfs)	M1 A1 2	or ${}^{12}C_2 / {}^{22}C_2$
(ii)	$\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13}$ or $\frac{8}{65}$ oe $\times 3$ oe $= \frac{24}{65}$ or 0.369 (3 sfs)	M1 M1 A1 3	Numerators any order ${}^7C_2 \times {}^8C_1$:M1 3 x prod any 3 probs (any C or P) ${}^{15}C_3$:M1 (dep <1) $1 - (\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} + 3 \times \frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13})$: M2 one prod omitted or wrong: M1
(iii)	$\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe $x^2 - x - 132 = 0$ or $x(x-1) = 132$ $(x-12)(x+11) = 0$ or $x = \frac{1 \pm \sqrt{(1^2 - 4 \times (-132))}}{2}$ No. of Ys = 12	M1 A1 M1 A1 4	not $\frac{x}{45} \times \frac{x}{44} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x}{45} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x-1}{45} = \frac{1}{15}$ oe ft 3-term QE for M1 condone signs interchanged allow one sign error Not $x = 12$ or -11 ans 12 from less wking, eg $12 \times 11 = 132$ or T & I: full mks Some incorrect methods: $\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15}$ oe M1 $x^2 + x = 132$ A0 $x = 11$ M1A0 $12 \times 11 = 132$ M1A1M1 $x = 12$ and (or "or") 11 A0 NB 12 from eg 12.3 rounded, check method
Total		9	

6(i)(a)	256	B1 1	(i)(b) & (ii)(abc): ISW ie if correct seen, ignore extras
(b)	Total unknown or totals poss diff or Y13 may be smaller or similar or size of pie chart may differ	B1 1	pie chart shows only proportions oe or no. of students per degree may differ not "no. of F may be less" not "Y13 may be larger"
(ii)(a)	B&W does not show frequencies oe	B1 1	or B&W shows spread or shows mks or M lger range
(b)	F generally higher or median higher F higher on average or F better mks F IQR is above M IQR F more compact M wide(r) range or gter IQR or gter variation or gter variance or more spread or less consistent M evenly spread or F skewed	B1 B1 2	1 mk about overall standard, based on median or on F's IQR being "higher" 1 mk about spread (or range or IQR) or about skewness. must be overall, not indiv mks must be comparison, not just figures Examples: not F higher mean not M have hiest and lowest mks condone F +ve skew
(c)	<u>Advantage:</u> B&W shows med or Qs or IQR or range or hiest & lowest or key values <u>Disadvantage:</u> B&W loses info' B&W shows less info' B&W not show freqs B&W not show mode B&W: outlier can give false impression hist shows more info hist shows freqs or fds hist shows modal class (allow mode) hist shows distribution better can calc mean from hist	B1 B1 2	not B&W shows skewness not B&W shows info at a glance not B&W easier to compare data sets not B&W shows mean not B&W shows spread not B&W easier to calculate or easier to read not B&W does not give indiv (or raw) data not B&W does not show mean not hist shows freq for each mark not hist shows all the results not hist shows total allow adv of hist as disadv of B&W
(iii)	102 x 51 + 26 x 59 ÷ 128 = 52.6 (3 sfs)	M1 M1dep A1 3	or 5202 + 1534 or 6736
Total		10	

7(i)	Geo stated $0.7^3 \times 0.3$ $\frac{1029}{10000}$ oe or 0.103 (3 sfs)	M1 M1 A1 3	or implied by $0.7^7 \times 0.3$ or $0.3^7 \times 0.7$ Allow $0.7^4 \times 0.3$
(ii)	0.7^6 alone = 0.118 (3 sfs)	M1 A1 2	$1 - (0.3 + 0.3 \times 0.7 + \dots + 0.3 \times 0.7^5)$ not $1 - 0.7^6$
(iii)	0.7^9 $1 - 0.7^9$ 0.960 (3 sfs)	M1 M1 A1 3	not 0.3×0.7^9 allow $1 - 0.7^{10}$ or 0.972 for M1 allow 0.96, if no incorrect wking seen $0.3 + 0.7 \times 0.3 + \dots + 0.7^8 \times 0.3$: M2 1 term omitted or wrong or "correct" extra: M1 or implied by table or ${}^n C_r$ or $0.7^3 \times 0.3^2$ or 0.0309
(iv)	Bin stated ${}^5 C_2 \times 0.7^3 \times 0.3^2$ or 0.8369 – 0.5282 = 0.3087 or 0.309 (3 sfs)	M1 M1 A1 3	
Total		11	
8(i)	$168.6 - \frac{88 \times 16.4}{8}$ $\sqrt{\left(1136 - \frac{88^2}{8}\right)\left(34.52 - \frac{16.4^2}{8}\right)}$ = -0.960 (3 sfs)	M2 A1 3	$\left(= \frac{-11.8}{\sqrt{168 \times 0.9}}\right)$ M1: correct subst in any correct S formula M2: correct substn in any correct r formula allow -0.96, if no incorrect wking seen
(ii)	must refer to, or imply, external constraint on x e.g x is controlled or values of x fixed or chosen allow x is fixed	B1 1	not x is not random not x affects y not x not affected by y not x goes up same amount each time not charge affects no. of vehicles not x not being measured
(iii)	$168.6 - \frac{88 \times 16.4}{8}$ $\frac{1136 - \frac{88^2}{8}}{8}$ = -0.0702 (3 sfs) or $^{-59/840}$ or $^{-11.8/168}$ $y - \frac{16.4}{8} = \text{"-0.0702"}(x - \frac{88}{8})$ $y = -0.07x + 2.8$ or better	M1 A1 M1 A1 4	ft their S_{xy} and S_{xx} incl $^{168.6/1136}$ if used in (i) or -0.07 if no incorrect wking or $a = \frac{16.4}{8} - (\text{"-0.0702"}) \times \frac{88}{8}$ or $^{2371/840}$ oe eg $y = \frac{-59}{840}x + \frac{2371}{840}$
(iv)(a)	"-0.07" $\times 20 + \text{"2.8"}$ = 1.4(2) million (2 sfs)	M1 A1 2	no ft
(b)	r close to -1 or corr'n is high just outside given data, so reliable	B1 B1 2	or good corr'n or pts close to line but not if "close to -1, hence unreliable" if r low in (i), ft: " r low" or "poor corr'n" etc or outside given data so unreliable not "reliable as follows trend" not "reliable as follows average" no ft from (iv)(a)
(v)	y on x x is indep	B1 B1 2	or x controlled or y depends on x or y not indep dep on not " x on y " r close to -1 so makes little difference: B2
Total		14	

4733 Probability & Statistics 2

General: Conclusions to hypothesis tests must acknowledge uncertainty. Thus “time is unchanged” is A0. Similarly, “Significant evidence that time is unchanged” is also A0.

1	(i)	Biased in favour of those with strong political interest	B2	2	“Biased”, “unrepresentative”, “not indept” or equiv [but <i>not</i> “not random”] stated, with sensible reason. [SR: partial answer, B1]
	(ii)	Obtain list of all pupils Allocate numbers sequentially Choose using random numbers	B1 B1 B1	3	List, can be implied; number serially or randomly, not just “number pupils” Select consistently with method of numbering, not just “select randomly” [SR: systematic: List B1, every n^{th} B1, random start B1] [SR: names in a hat: B2]
2	(i)	$\Phi\left(\frac{24-30}{12}\right) - \Phi\left(\frac{20-30}{12}\right)$ $= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976) = \mathbf{0.1061}$	M1 A1 M1 A1	4	Standardise one, allow $\sqrt{12}$, 12^2 , \sqrt{n} Both standardisations correct, allow cc here Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
	(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	2	Any comment implying not symmetric Conclude “not good model” [Partial answer: B1]
3	(i)	$H_0 : \mu = 28$ $H_1 : \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38] $z = \frac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare -1.645 , or 0.0547 with 0.05	B2 M1 M1 A1 B1		Both hypotheses correctly stated; one error, allow wrong or no letter, but not x or t or \bar{x} , B1 Multiply 37.05 or $\sqrt{37.05}$ by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$ Standardise with \sqrt{n} , allow $\sqrt{\text{errors}}$, cc, + Correct z , a.r.t. -1.60 , or $p \in [0.0547, 0.0548]$ Explicit comparison of z with -1.645 or p with 0.05
	β	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare 26.44 with 26.40	M1 B1 A1 $\sqrt{}$		Allow “ \pm ”, $\sqrt{\text{errors}}$, cc, ignore other tail $z = 1.645$ in CV expression, and compare 26.44 CV, $\sqrt{\text{}}$ on their z , rounding to 3 SF correct
		Do not reject H_0 [can be implied] Insufficient evidence that time taken has changed.	M1 A1 $\sqrt{}$	8	Needs \sqrt{n} , correct method & comparison, <i>not</i> $\mu = 26.44$ Conclusion interpreted in context, $\sqrt{\text{}}$ on z ,
4	(i)	$\frac{53-50}{\sigma/\sqrt{10}} < 2.326$ $\sigma > \mathbf{4.08}$ AG [Allow \geq]	M1 A1 B1 A1	4	Standardise with 10 or $\sqrt{10}$ and Φ^{-1} Both sides same sign, $\sqrt{10}$, don’t worry about $<$ 2.326 or 2.33 seen Convincingly obtain $\sigma > 4.08$ to 3 SF, one other step [SR: Substitution: standardise & substitute 4.08 M1; 0.0101 A1; 4.07 or 4.075 tried, M1; full justification A1]
	(ii)	$P(\text{Type I}) = 0.01$ used, e.g. Geo(0.01) $0.99^4 \times 0.01$ $= \mathbf{0.0096}$	M1 M1 A1	3	Not enough merely to state $p = 0.01$ $p^4 \times q$ Answer, a.r.t. 0.0096
5	(i)	$\int_{-1}^1 \frac{3}{4}(x^2 - x^4)dx = \frac{3}{4} \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_{-1}^1$ [= 1/5] $1/5 - 0^2$ $= \mathbf{1/5}$	M1 A1 B1 A1	4	Attempt $\int_{-1}^1 x^2 f(x) dx$ Correct indefinite integral Mean 0 clearly indicated Answer $1/5$ or a.r.t. 0.200 , don’t need $\mu = 0$
	(ii)	 (a) (b) Areas equal, more spread out, so g_{max} lower (c) W greater as more spread out	B1 M1 A1 B1 dep depB1	5	Correct graph, don’t need $f(x)$ as well. Don’t allow if graph goes further below axis than “pips”. Don’t worry too much about exact shape Mention areas or total probability Convincing argument, not just “flatter” W greater... ...with convincing reason

6	(a)	Po(2.375) $e^{-2.375} \left(\frac{2.375^3}{3!} + \frac{2.375^4}{4!} \right) [= 0.2079 + 0.1233]$ $= \mathbf{0.3310}$	M1 M1 A1 A1	4	Po(19/8) stated or implied One correct Poisson formula, <i>not</i> tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]
	(b) (i)	n large OR $n > 50$ p small OR $np < 5$	B1 B1	2	Or equivalent [Allow \leq and \geq throughout] Or equivalent, e.g. $np \approx npq$, or $p < 0.1$ [Treat " $np < 5$, $npq < 5$ " as single wrong statement]
	(ii)	$B(108, \frac{1}{36})$ $\approx \text{Po}(3)$ $1 - P(\leq 3) = 1 - 0.6472$ $= \mathbf{0.3528}$	M1 M1 A1 M1 A1	5	Correct binomial distribution stated or implied Po(np), $\sqrt{\quad}$ on their n, p Po(3) Use Po tables, "1 -", or correct formula, ± 1 term, e.g. 0.1847; a.r.t. 0.353, allow from exact Binomial
7	(i)	Dropped catches must occur independently of one another and at constant average rate	B1 B1	2	"independently", in context, allow "random" "Constant average rate", in context ["Singly" doesn't gain B1]
	(ii)	Use: "Reject H_0 when correct" Po(10) $P(\geq 16) = 1 - P(\leq 15) = 1 - 0.9513$ Probability 0.0487	M1 M1 M1		Find $P(\geq r)$ where $r > \lambda$, e.g. $P(\geq 6)$ from Po(2) Po(10) stated or implied [can be recovered in (iii)] Seek biggest prob < 0.05 , e.g. 0.0835 or 0.0166, allow 0.0293 but no other LH tail Answer in range [0.0487, 0.0488], cwd, cwo
		(iii)	$H_0 : \lambda = 10$ or 2 [or μ] $H_1 : \lambda > 10$ or 2 [or μ] $\alpha : P(\geq 14) = 1 - 0.8645 = 0.1355 > 0.05$ $\beta : \text{Critical region } r \geq 16, p = 0.0487$ Compare $r = 14$	B2 A1 B1 A1 B1	
		Do not reject H_0 [can be implied] Insufficient evidence of an increase in the number of dropped catches	M1 A1	10	Method correct, $\sqrt{\quad}$ on p , must be upper tail and " \geq " Conclusion interpreted in context [SR: $P(\leq 14) = 0.9165 < 0.95$: (B2 M1) A0 B1 M0A0; same for $P(> 14)$ or $P(= 14)$] [SR: $N(10, 10)$: (ii) 0.05 M0. (iii) (B2) M1 A0 B1 M0A0]
	8	(i)	$H_0 : p = 0.4$ or $\mu = 4.8$ $H_1 : p > 0.4$ or $\mu > 4.8$ B(12, 0.4) $P(\geq 9) = 1 - 0.9847 = 0.0153 < 0.05$ Reject H_0 [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2 M1 A1 B1 M1 A1	7
(ii)		$N(160, 96)$ $\frac{(x-0.5)-160}{\sqrt{96}} = 1.645$ Solve to find x [= 176.6] Minimum value is 177	B1 B1 M1 A1 B1 M1 A1	7	Normal, mean 160 Variance (or SD) 96 [96/400: B2M0] Standardise unknown with np and \sqrt{npq} or npq , & equate to Φ^{-1} ; $\sqrt{96}$ and signs correct, ignore cc RHS = 1.645 Solve [implied by 177 or 176.6 or 176.1] 177 only, from 176.6, CWO [cc error: 6 ex 7]

Marks can be awarded in either part

4734 Probability & Statistics 3

1 (i) $\frac{1}{99}(6115.04 - \frac{761.2^2}{100})$ M1 AEF
 =3.240 A1 2

(ii) $761.2/100 \pm z\sqrt{(3.24/100)}$ M1 $z = 1.282, 1.645, \text{ or } 1.96$
 $z = 1.96$ B1
 (7.26, 7.96) A1 3 Allow from $\sigma^2 = 3.21$; allow 7.97 but not from wrong σ . Allow 4 or 5 SF but no more.

(iii) None necessary, since sample size large enough for sample mean to have a normal distribution OR: None necessary, n large enough for Central Limit theorem to apply
 B1 1
 [6]

2 $(\bar{x} - 12.6) / \sqrt{0.1195 / 10}$ M1 Any variable, correct mean, /10, ignore z
 A1 All correct
 B1
 M1 Allow any symbol (<, >, =)
 A1 5 Allow > ; 12.7 or 12.8 No z seen
 [5]

3(i) Choice of newspaper is independent of level of income B1 1 Or equivalent

(ii) Use $df=4$ B1 May be implied by 13.28 seen or 0.0152
 EITHER: CV 13.28, from $df=4$ or sig. level M1 From tables
 Largest significance level is 1% B1 Accept 0.01
 OR: Use $P(\chi^2 > 12.32)$ Use of calculator
 Largest significance level is 1.52% B2 3 Accept 0.0152
 [4]
 SR: from $df=6$: CV 12.59 used ; SL=5% :
 B0M1B1

4(i) $\int_0^1 \frac{4}{3}x^3 dx + \int_1^2 \frac{4}{3x^3} dx$ Limits seen anywhere M1 For both integrals OR $1 - \int_2^\infty \frac{4}{3x^3} dx$
 $\left[\frac{x^4}{3} \right]_0^1 + \left[-\frac{2}{3x^2} \right]_1^2$ A1 For both OR $1 - \left[-\frac{2}{3x^2} \right]_2^\infty$
 $\frac{5}{6}$ A1 3

(ii) EITHER: $\int_0^1 \frac{4}{3}x^3 dx = \frac{1}{3}$ M1
 $< \frac{1}{2}$ A1
 Median must exceed 1 A1
 OR: M1 Attempt to find median
 $m = \sqrt{(4/3)}$ A1 M0 for $1.5^{1/4}$
 > 1 AG A1 3 Accept 1.15..

(iii)	$\int_0^1 \frac{4}{3} x^4 dx + \int_1^{\infty} \frac{4}{3x^2} dx$ $[4x^5/15] + [-4/(3x)]$ 1.6	M1	Correct form for at least one integral
		B1	Both integrals correct without limits
		A1	3 AEF

--			
(iv)	$E(X^2) = \dots + \int_1^{\infty} \frac{4}{3x} dx$ Second integral = $\left[\frac{4}{3} \ln x \right]_1^{\infty}$ This is not finite, (so variance not finite)	M1	For second integral
		A1	
		A1	3 AEF
			[12]

5 (i)	Justify a relevant Poisson approximation $E(A) = 75 \times 0.022 (=1.65)$, $E(B) = 90 \times 0.025 (=2.25)$ Sum of two independent Poisson variables X has a Poisson distribution Mean $m = 3.9$	M1	Using $n > 50$ or n large; $np < 5$ or p small (< 0.1)
		B1B1	or $np \approx npq$
		A1	
		B1	5 Accept Po(3.9)

(ii)	$1 - P(\leq 5)$ 0.1994	M1	Or From Po(m) Accept ≤ 4 ; OR Exact $1 -$ sum of at least 5 correct terms
		A1	2 From calculator or tables, art 0.20
			[7]

6 (i)	Use $p_s \pm z s$ $z = 2.326$ $s = \sqrt{(0.12 \times 0.88/50)}$ $(0.013, 0.227)$ Allow limits if penalised in Q1	M1	
		B1	
		A1	Or /49
		A1	4 Or (0.012, 0.228) from 49

(ii)	$z(0.12 \times 0.88/n)^{1/2}$ < 0.05 Solve to obtain $n > 228.5$ $n \approx 229$ or 230	M1	Any z
		A1	Allow =
		M1	Must contain \sqrt{n}
		A1	Accept =
		A1	5 Must be integer
			[9]

7 (i)	Each population of test scores should have normal distributions with equal variances	B1	OR: Variances equal and normal distns
		B1	Context
		B1	2

(ii)	EITHER: Cannot test for normality from data OR: Sample variances are close enough to accept population variances equal		Not variances are not equal
		B1	1



<p>(iii) $H_0: \mu_B = \mu_G, H_1: \mu_B > \mu_G$ $s^2 = (23 \times 86.79 + 17 \times 93.01) / 40$ $= 89.4335$ $t = (1238.4 / 18 - 1526.8 / 24) / [s^2(18^{-1} + 24^{-1})]^{1/2}$ $= 1.758$ Use CV of 1.684 $1.758 > 1.684$ Reject H_0 and accept there is sufficient evidence at the 5% significance level that teenage boys worry more, on average than teenage girls.</p>	<p>B1 For both. No other variables. Allow words M1 Finding pooled estimate of variance A1 May be implied by later value of t M1 With pooled estimate of variance A1 All correct A1 art 1.76, or - B1 Consistent M1 Compare correctly with their CV (t value) Not assertive A1√ 9 Ft on their 1.758 SR: Using $s^2 = 93.01/18 + 86.79/24$: B1M0A0M1A0A1 (for 1.749) B1M1 (from 1.645 or 1.684) A1 Max 6/9</p>
12]	
<p>8 (i) $\Sigma x/f/80 = 1.9$ AG $\Sigma x^2/f/80 - 1.9^2$ 1.365 or 1.382</p>	<p>B1 With evidence M1 Or $\times 80/79$ A1 3</p>
<p>(ii) Poisson distribution requires equal mean and variance EITHER: No, mean and variance differ significantly OR: Yes, indicated by sample statistics taking into account sampling error</p>	<p>B1 May be indicated B1 2</p>
<p>(iii) $e^{-1.9} 1.9^3 / 3!$ $\times 80$</p>	<p>B1 Or from tables B1 2</p>
<p>(iv) Considering sample as random selection of all similar matches H_0: Poisson suitable model Combine last two cells $0.97^2 / 11.97 + 7.73^2 / 22.73 + 11.40^2 / 21.60$ $+ 2.32^2 / 13.68 + 5.02^2 / 10.02$ $= 11.63$ CV 7.815 $11.63 > 7.815$ There is sufficient evidence that a Poisson distribution is not a suitable model confirming (or not) the answer to part (ii)</p>	<p>B1 B1 M1 Any two correct A1 All correct A1 art 11.6 B1 *dep OR $p = 0.00875$ M1dep* OR $0.00875 < 0.05$ A1√ 8 Ft (ii) SR: If last cells not combined: $\chi^2 = 12.3$ M1A1A1 CV=9.448 or $p = 0.0152$, B1*dep the M1dep*</p>
<p>(v) E-values or probabilities would change df would increase by 1</p>	<p>B1 Or other valid observation B1 2 Or CV would change [17]</p>

4735 Statistics 4

1 (i)	Use $P(A) + P(B) - P(A \cap B) \leq 1, P(A \cap B) = 0$	B1	1	AEF

(ii)	Use $P(A B) = P(A \cap B) / P(B)$ Use $P(A \cap B) = 0$ with argument with $x \neq 0$	M1 A1	2	AEF e.g. Independent if $(A \cap B) = P(A)P(B) = x^2$, $P(A \cap B) = 0, x \neq 0$, so A and B are not indep.

(iii)	Use $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$ Use $P(A \cap B) = 0; P(A \cap B \cap C) = 0$ $P(B \cap C) = 2x^2; P(C \cap A) = 2x^2$ Substitute and obtain required result	M1 A1 A1 AG	4 (7)	Or equivalent. Allow one sign error For both For both

2 (i)	Wilcoxon test requires a symmetric distribution not supported by the diagram	B1	1	Or equivalent

(ii)	$H_0: m = 1.80, H_1: m > 1.80$ Use sign test Number exceeding 1.8 = 20 Use $B(30, 0.5), P(\geq 20)$ Or $P(\leq 10)$ 0.0494 Compare with 0.05 correctly 2.008 Conclude there is significant evidence that the median time exceeds 1.80 sec	M1 A1 M1 A1 M1 A1√	7 (8)	B1 Needs "population median" if words OR: 1.645 if $N(15, 7.5), z = 1.643, 1.816$, used; OR CR ($X \geq 20$) fit p or z

3 (i)	Marginal distribution of X x 0 1 2 3 p 0.27 0.23 0.32 0.18 $1 \times 0.23 + 2 \times 0.32 + 3 \times 0.18$ $= 1.41$	B1 M1 A1	3	

(ii)	$P(Y > X) = 0.08 + 0.05 + 0.03 + 0.08 + 0.06 + 0.07$ $= 0.37$	M1 A1	2	

(iii)	Use $P(Y > X \cap X > 0) / P(X > 0)$ $P(X > 0) = 0.73$ $P(Y > X \cap X > 0) = 0.08 + 0.06 + 0.07$ $21/73$	M1 A1 A1 A1	4	From marginal distribution AEF

(iv)	The director cannot conclude independence from cov. So director's conclusion incorrect. OR: Eg $P(X=0 \cap Y=0) = 0.11$, $P(X=0)P(Y=0) = 0.27 \times 0.29 \neq P(X=0 \cap Y=0)$	M1 A1 M1 A1	2 (11)	Idea that independence implies cov = 0 but not the reverse

4 (i)	Variances seem not to be equal	B1	1	

(ii)	$H_0: m_M = m_A, H_1: m_M \neq m_A$ “average” $R_m = 40, m(m+n+1) - R_m = 72$ $W = 40$ CR: $W \leq 38$ 40 not in CR, so do not reject H_0 Insufficient evidence that median times differ	B1 M1 A1 B1 M1 A1	6 (7) Both hypotheses, AEF. Not Both found A0 if no or wrong 72 Or equivalent In context. B1 if no M1 but conclusion correct Allow average here	

5 (i)	$a+b = \frac{3}{4}$ $M'(0) = \frac{3^3}{8}$ $\frac{1}{2} + 3a + 4b = \frac{3^3}{8}$ Solve simultaneously $a = \frac{1}{8}$ AG $b = \frac{5}{8}$	B1 M1 A1 M1 A1 A1	6 From $M(0)=1$ AEF Elimination or substitution	

(ii)	$M''(t) = e^{2t} + \frac{9}{8}e^{3t} + 10e^{4t}$ $M''(0) - (M'(0))^2$ $\frac{97}{8} - (\frac{3^3}{8})^2 ; \frac{47}{64}$	B1 M1 A1A1	4	

(iii)	$x = 2, 3, 4$	B1	1 (11)	

6 (i)	$P(Y > y) = 1 - F(y)$ $= \frac{a^3}{y^3}$ $P(S > s) = P(\text{all 3 values } > s) = (a/s)^9$ AG $f(s) = d/ds(1 - (a/s)^9)$ $= \begin{cases} 9 \frac{a^9}{s^{10}} & s \geq a \\ 0 & s < a \end{cases}$	M1 A1 A1 M1 A1	5 Allow any variables	

(ii)	$\int_a^\infty \frac{a^9}{s^9} ds$ $= 9a/8$ S not unbiased since this not equal to a $T_1 = 8S/9$	M1 A1 M1 B1√	4	Ft $E(S)$

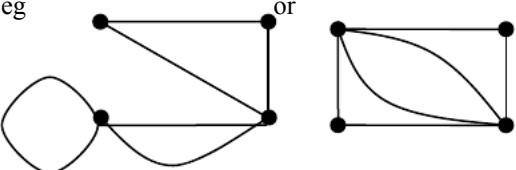
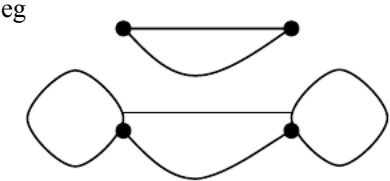
(iii)	$\text{Var}(T_1) = a^2/63, \text{Var } T_2 = a^2/9$ $\text{Var}(T_1) < \text{Var}(T_2), T_1$ is more efficient	M1 A1 for both A1√	3	Correct method Comparison, completion.. √ one variance correct with same dimensions

(iv)	$t_1 = 4.0, t_2 = 5.4$ From data $a \leq 4.5$ and $t_2 > 4.5$	B1 B1B1	3 (15)	Both AEF

7 (i)	$G(1) = 1$ $a = 2$	M1 A1	2
<hr/>			
(ii)	$(1+2t)/(4-t) = c(1+2t)(1-\frac{1}{4}t)^{-1}$ $= \frac{1}{4}(1+2t)(1 + \frac{1}{4}t + (\frac{1}{4}t)^2 + \dots)$ Coefficient of $t^3 = \frac{1}{4}[(\frac{1}{4})^3 + 2(\frac{1}{4})^2]$ $= \frac{9}{256}$	M1 A1 M1√	$c = \frac{1}{4}$ or 4 With 2 terms from previous line A1 4
<hr/>			
(iii)	$H(t) = \left(\frac{1+2t}{4-t}\right)^3$ $H'(t) = 3\left(\frac{1+2t}{4-t}\right)^2 \left[\frac{2(4-t)+1+2t}{(4-t)^2}\right]$ $E(Y) = H'(1)$ $= 3$	B1 M1A1 M1 A1	5
<hr/>			
(iv)	$H(1) = p_0 + p_1 + p_2 + p_3 + p_4 + \dots = 1$ $H(-1) = p_0 - p_1 + p_2 - p_3 + p_4 - \dots = -\frac{1}{125}$ Add: $2(p_0 + p_2 + p_4 + \dots) = 1 - \frac{1}{125}$ $\frac{1}{2}(1 - \frac{1}{125})$ AG	M1 A1	With sufficient detail 2 (13)

4736 Decision Mathematics 1

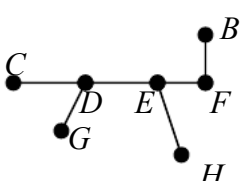
1	(i)	Biggest/largest/last number (only) (Not showing effect on a specific list)	B1	Accept bubbling to left unless inconsistent with part (ii): Smallest/first number	[1]
	(ii)	2 1 3 4 5 horizontally or vertically (may see individual comparisons/swaps) [For reference: original list was 3 2 1 5 4] 4 comparisons and 3 swaps (both correct)	M1 A1	Or bubbling to left: 1 3 2 4 5 Watch out for shuttle sort used If not stated, assume that comparisons come first	[2]
	(iii)	1 2 3 4 5 One (more pass after this)	M1 A1	FT from their first pass with their bubbling if possible Watch out for 'One swap (in 2 nd pass)'	[2]
	(iv)	$(3000 \div 500)^2 \times 0.2$ = 7.2 seconds	M1 A1	$6^2 \times 0.2$ or $8 \times 10^7 \times 9 \times 10^6$ or any equivalent calculation cao UNITS	[2]
Total = 7					

2	(i)	eg  - Graph is not simple - Two of the vertices are joined by two arcs (if appropriate) - It has a 'loop' (if appropriate) - For a simple graph each vertex must have order 3 or less	M1 A1 B1	A graph with four vertices of orders 2, 2, 4, 4 (ignore any vertex labels) A connected graph Recognition that their graph is not simple (although it is connected). Need not use the word 'simple'.	[3]
	(ii)	eg  Graph is not connected	M1 A1 B1	Any graph with four vertices of orders 2, 2, 4, 4 (that is topologically different from that in part (i)) A graph that is not connected Recognition in words that their graph is not connected	[3]
Total = 6					

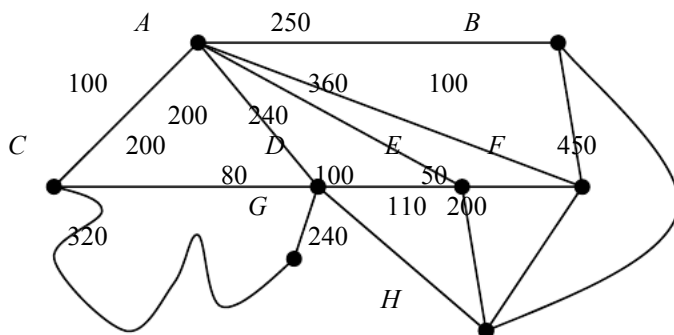
3	(i)	$y \leq x + 2$ $x + 2y \geq 6$ ($y \geq -\frac{1}{2}x + 3$) $2x + y \leq 12$ ($y \leq -2x + 12$)	M1 M1 M1 A1	Line $y = x + 2$ in any form Line $x + 2y = 6$ in any form Line $2x + y = 12$ in any form All inequalities correct	[4]
	(ii)	$x + 2y = 6$ and $y = x + 2 \Rightarrow (\frac{2}{3}, 2\frac{2}{3})$ $y + 2x = 12$ and $y = x + 2 \Rightarrow (3\frac{1}{3}, 5\frac{1}{3})$ $y + 2x = 12$ and $x + 2y = 6 \Rightarrow (6, 0)$	M1 A1 A1 B1	Follow through if possible Calculating from their lines or implied from either A mark $(\frac{2}{3}, \frac{8}{3})$ (art (0.7, 2.7)) $(\frac{10}{3}, \frac{16}{3})$ (art (3.3, 5.3)) $(6, 0)$ cao	[4]
	(iii)	$(\frac{2}{3}, 2\frac{2}{3}) \Rightarrow 11\frac{1}{3}$ $(3\frac{1}{3}, 5\frac{1}{3}) \Rightarrow 32\frac{2}{3}$ $(6, 0) \Rightarrow 30$ At optimum, $x = 3\frac{1}{3}$ and $y = 5\frac{1}{3}$ Maximum value = $32\frac{2}{3}$	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept $(3\frac{1}{3}, 5\frac{1}{3})$ identified (ft) $32\frac{2}{3}$ (air 32.6 to 32.7) (ft)	[3]
	(iv)	$5 \times 3\frac{1}{3} + k \times 5\frac{1}{3} \geq 5 \times 6 + k \times 0$ $\Rightarrow k \geq 2.5$	M1 M1 A1	$5 \times 3\frac{1}{3} + k \times 5\frac{1}{3}$ (ft) or implied $5 \times 6 + k \times 0$ or 30 or implied Greater than or equal to 2.5 (cao)	[3]
Total =					14

<p>4 (i)</p>	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>1</td><td>0</td></tr> <tr><td colspan="2"> </td></tr> </table> <p><i>A</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>4</td><td>5</td></tr> <tr><td>6</td><td>5</td></tr> </table> <p><i>B</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>5</td><td>6</td></tr> <tr><td>6</td><td> </td></tr> </table> <p><i>C</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>(9)</td><td>(16)</td></tr> <tr><td>16</td><td> </td></tr> </table> <p><i>F</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>7</td><td>12</td></tr> <tr><td>12</td><td> </td></tr> </table> <p><i>H</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>3</td><td>3</td></tr> <tr><td>4</td><td>3</td></tr> </table> <p><i>D</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>2</td><td>2</td></tr> <tr><td>2</td><td> </td></tr> </table> <p><i>E</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>6</td><td>10</td></tr> <tr><td>10</td><td> </td></tr> </table> <p><i>G</i></p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>(10)</td><td>(16)</td></tr> <tr><td>16</td><td> </td></tr> </table> <p><i>J</i></p> </div> <div style="text-align: center;"> <table border="1" style="margin-bottom: 10px;"> <tr><td>8</td><td>14</td></tr> <tr><td>14</td><td> </td></tr> </table> <p><i>K</i></p> </div> </div> <p style="margin-top: 20px;">Route = $A - E - B - G - H - K$ Length = 14 metres</p>	1	0			4	5	6	5	5	6	6		(9)	(16)	16		7	12	12		3	3	4	3	2	2	2		6	10	10		(10)	(16)	16		8	14	14		<p>M1 Both 6 and 5 shown at <i>B</i></p> <p>M1 All temporary labels correct including <i>F</i> and <i>J</i></p> <p>A1 No extra temporary labels</p> <p>B1 All permanent labels correct (may omit <i>F</i> and/or <i>J</i>) cao</p> <p>B1 Order of labelling correct (may omit <i>F</i> and/or <i>J</i>, may reverse <i>F</i> and <i>J</i>) cao</p> <p>B1 $A - E - B - G - H - K$ cao</p> <p>B1 14 cao</p>	<p>[7]</p>
1	0																																										
4	5																																										
6	5																																										
5	6																																										
6																																											
(9)	(16)																																										
16																																											
7	12																																										
12																																											
3	3																																										
4	3																																										
2	2																																										
2																																											
6	10																																										
10																																											
(10)	(16)																																										
16																																											
8	14																																										
14																																											
<p>(ii)</p>	<p>Without using <i>CJ</i>: Route = $A - E - B - G - F - J$ Length = 21 metres</p>	<p>B1 Follow through their (i)</p> <p>B1 $A - E - B - G - F - J$</p> <p>B1 21</p>	<p>[2]</p>																																								
<p>(iii)</p>	<p>More than 2 metres</p> <p>(Answer of 'more than 7 metres' or '7 metres' ⇒ M1, A0)</p>	<p>M1 2 (cao)</p> <p>A1 More than, or equivalent</p> <p>(Answer of 3 or ≥ 3 ⇒ SC1)</p>	<p>[2]</p>																																								
<p>Total = 11</p>																																											

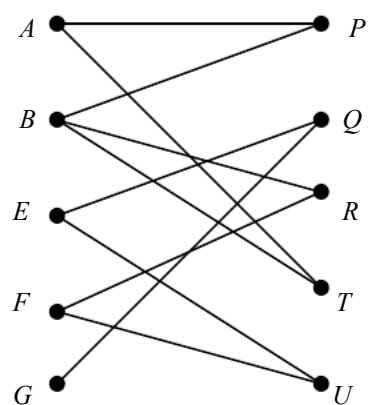
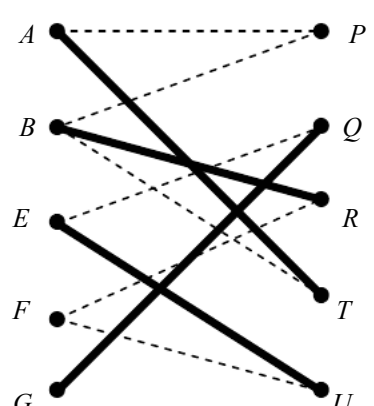
5	(i)	<table border="1"> <thead> <tr> <th></th> <th><i>E</i></th> <th><i>W</i></th> </tr> </thead> <tbody> <tr> <td><i>A</i></td> <td>x</td> <td>$3 - x$</td> </tr> <tr> <td><i>B</i></td> <td>y</td> <td>$3 - y$</td> </tr> <tr> <td><i>C</i></td> <td>$4 - x - y$</td> <td>$x + y - 1$</td> </tr> </tbody> </table>		<i>E</i>	<i>W</i>	<i>A</i>	x	$3 - x$	<i>B</i>	y	$3 - y$	<i>C</i>	$4 - x - y$	$x + y - 1$	B1 B1 B1	$AW = 3 - x$ $BW = 3 - y$ $CE = 4 - x - y$, in any form	[5]																							
			<i>E</i>	<i>W</i>																																				
		<i>A</i>	x	$3 - x$																																				
		<i>B</i>	y	$3 - y$																																				
		<i>C</i>	$4 - x - y$	$x + y - 1$																																				
<p>Total cost = $\pounds(250x + 250(3-x) + 200y + 140(3-y) + 300(4-x-y) + 280(x+y-1))$ $= \pounds(2090 - 20x + 40y)$ (AG)</p>	M1 A1	An appropriate calculation for their table Leading to given result																																						
(ii)	$2090 - 20x + 40y \leq 2150$ $\Rightarrow -20x + 40y \leq 60$ $\Rightarrow -x + 2y \leq 3$ (AG)	B1	Showing where the given inequality comes from	[1]																																				
(iii)	$50(3-x) + 40(3-y) + 60(x+y-1)$ $= 210 + 10x + 20y$ So need to maximise $x + 2y$ (AG)	M1 A1	Follow through their table Correct expression $210 + 10x + 20y$	[2]																																				
(iv)	<table border="1"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>s</i></th> <th><i>t</i></th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-1</td> <td>-2</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>-1</td> <td>2</td> <td>1</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> <td>3</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	-	1	-1	-2	0	0	0	0	-1	2	1	0	3	0	1	1	0	1	3	B1 B1	Rows and columns may be in any order -1 -2 in objective row Constraint rows correct	[2]												
<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	-																																			
1	-1	-2	0	0	0																																			
0	-1	2	1	0	3																																			
0	1	1	0	1	3																																			
(v)	<p>Pivot on the 2 in the <i>y</i> column</p> <table border="1"> <tbody> <tr> <td>1</td> <td>-2</td> <td>0</td> <td>1</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td>-0.5</td> <td>1</td> <td>0.5</td> <td>0</td> <td>1.5</td> </tr> <tr> <td>0</td> <td>1.5</td> <td>0</td> <td>-0.5</td> <td>1</td> <td>1.5</td> </tr> </tbody> </table> <p>Pivot on 1.5 in the <i>x</i> column</p> <table border="1"> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>$\frac{1}{3}$</td> <td>$1\frac{1}{3}$</td> <td>5</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>$\frac{1}{3}$</td> <td>$\frac{1}{3}$</td> <td>2</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>$-\frac{1}{3}$</td> <td>$\frac{2}{3}$</td> <td>1</td> </tr> </tbody> </table> <p>$x = 1, y = 2$</p>	1	-2	0	1	0	3	0	-0.5	1	0.5	0	1.5	0	1.5	0	-0.5	1	1.5	1	0	0	$\frac{1}{3}$	$1\frac{1}{3}$	5	0	0	1	$\frac{1}{3}$	$\frac{1}{3}$	2	0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	1	B1 M1 A1 M1 A1 B1	Correct choice of pivot from <i>y</i> column Follow through their tableau and valid pivot if possible Pivot row correct Other rows correct Correct choice of pivot Follow through their tableau and valid pivot if possible Correct tableau Correct answer only	[6]
1	-2	0	1	0	3																																			
0	-0.5	1	0.5	0	1.5																																			
0	1.5	0	-0.5	1	1.5																																			
1	0	0	$\frac{1}{3}$	$1\frac{1}{3}$	5																																			
0	0	1	$\frac{1}{3}$	$\frac{1}{3}$	2																																			
0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	1																																			
Total = 16																																								

6	(a)(i)	Route Inspection (problem)	B1	Or Chinese postman (problem)	[1]
	(ii)	<p>Odd nodes are A, B, C and D</p> <p>$AB = 250$ $AC = 100$ $AD = 200$ $CD = 200$ $BD = 250$ $BC = 350$ 450 350 550</p> <p>Repeat AC and $BFED = 350$ Length of shortest route = 3350 metres</p>	B1 M1 A1 B1	<p>Identifying odd nodes (may be implied from working)</p> <p>Pairing odd nodes (all three pairings considered)</p> <p>M mark may not be implied</p> <p>350 as minimum</p> <p>3350 m or 3.35 km UNITS</p>	[4]
	(iii)	<p>C is an odd node, so we can end at another odd node.</p> <p>$AB = 250$ $AD = 200$ $BD = 250$ Repeat $AD = 200$ Length of route = 3200 metres Route ends at B</p>	M1 A1 B1	<p>Working need not be seen</p> <p>May be implied from answer</p> <p>3200</p> <p>B</p>	[3]
	(b)(i)	<p>$D - G - C - A - E - F - B - H - D$</p> <p>1580 metres $A - C - D - G$ then method stalls</p>	M1 A1 B1	<p>Correct cycle</p> <p>If drawn then arcs must be directed</p> <p>1580</p> <p>Identifying the stall</p>	[3]
	(ii)	<p>$BF = 100$ $FE = 50$ $ED = 100$ $DG = 80$ $EH = 110$ $DC = 200$</p>  <p>Order of adding nodes: $B F E D G H C$ Total weight of tree = 640 metres</p>	M1 A1 B1 A1 B1	<p>Use of Prim's algorithm to build tree (e.g. an attempt at list of arcs or order of adding vertices). NOT Kruskal</p> <p>Correct arcs chosen (listed or seen on tree)</p> <p>A correct tree with vertices labelled</p> <p>Order stated or clearly implied</p> <p>640</p>	[5]
	(iii)	<p>Lower bound = $640 + 100 + 200 = 940$ $940 \text{ metres} \leq \text{shortest tour} \leq 1580 \text{ metres}$</p>	M1 A1	<p>$300 + \text{weight of their tree}$ $\text{their } 940 \leq \text{length} \leq \text{their } 1580$ (condone use of $<$ here)</p>	[2]
Total = 18					

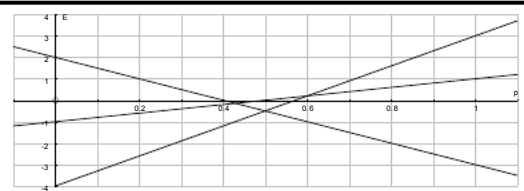
For reference:



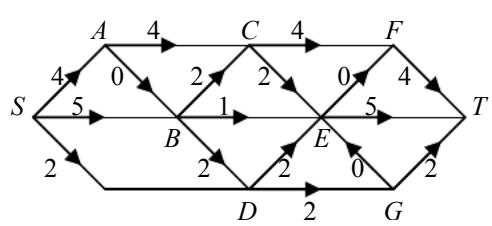
4737 Decision Mathematics 2

1(a)	(i)		B1	A correct bipartite graph	[1]																																			
	(ii)		B1	A second bipartite graph showing the incomplete matching correctly	[1]																																			
	(iii)	<p>$F - R - B - P$</p> <p>$A = T \quad B = P \quad E = U \quad F = R \quad G = Q$</p>	B1	This path in any reasonable form																																				
	(iv)	<p>$A = P \quad B = T \quad E = U \quad F = R \quad G = Q$</p>	B1	This complete matching	[2]																																			
(b)	(i)	<p>Hungarian algorithm finds the minimum cost matching, subtract from 10 to convert a maximising problem into a minimising problem.</p> <p>Column X is a dummy column (dummy task) to make the table square</p>	B1	An appropriate reference to maximising/minimising																																				
	(ii)	<table border="1" data-bbox="311 1489 845 1680"> <thead> <tr> <th></th> <th>C</th> <th>D</th> <th>L</th> <th>S</th> <th>X</th> </tr> </thead> <tbody> <tr> <th>H</th> <td>1</td> <td>2</td> <td>4</td> <td>4</td> <td>10</td> </tr> <tr> <th>I</th> <td>2</td> <td>4</td> <td>7</td> <td>6</td> <td>10</td> </tr> <tr> <th>J</th> <td>4</td> <td>6</td> <td>5</td> <td>9</td> <td>10</td> </tr> <tr> <th>K</th> <td>3</td> <td>8</td> <td>7</td> <td>7</td> <td>10</td> </tr> <tr> <th>N</th> <td>3</td> <td>7</td> <td>7</td> <td>5</td> <td>10</td> </tr> </tbody> </table>		C	D	L	S	X	H	1	2	4	4	10	I	2	4	7	6	10	J	4	6	5	9	10	K	3	8	7	7	10	N	3	7	7	5	10	B1	'Dummy' or 'square table' or equivalent
	C	D	L	S	X																																			
H	1	2	4	4	10																																			
I	2	4	7	6	10																																			
J	4	6	5	9	10																																			
K	3	8	7	7	10																																			
N	3	7	7	5	10																																			
					For reference only																																			

	<p>Reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>2</td><td>3</td><td>2</td><td>0</td></tr> <tr><td>3</td><td>4</td><td>1</td><td>5</td><td>0</td></tr> <tr><td>2</td><td>6</td><td>3</td><td>3</td><td>0</td></tr> <tr><td>2</td><td>5</td><td>3</td><td>1</td><td>0</td></tr> </table> <p>Rows are already reduced</p> <p>Augment by 1</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>2</td><td>1</td><td>0</td></tr> <tr><td>2</td><td>3</td><td>0</td><td>4</td><td>0</td></tr> <tr><td>1</td><td>5</td><td>2</td><td>2</td><td>0</td></tr> <tr><td>1</td><td>4</td><td>2</td><td>0</td><td>0</td></tr> </table> <p>$H = D$ Harry is the director $I = C$ Iannos operates the camera $J = L$ Jack is in charge of lighting $N = S$ Nadia is in charge of sound (and Kerry is not used)</p> <p>Total score = $(10-2) + (10-2) + (10-5) + (10-5) + (10-10)$ $= 26$</p>	0	0	0	0	0	1	2	3	2	0	3	4	1	5	0	2	6	3	3	0	2	5	3	1	0	0	0	0	0	1	0	1	2	1	0	2	3	0	4	0	1	5	2	2	0	1	4	2	0	0	<p>M1 Either reducing columns or reducing rows of 5×5 matrix</p> <p>A1 This reduced matrix Correct answer only</p> <p>M1 A reasonable attempt to augment</p> <p>A1 This final matrix Correct answer only</p> <p>B1 This matching, indicated in any way</p> <p>M1 A reasonable attempt, 14 or 24 \Rightarrow M1, A0</p>	<p>[7]</p>																																							
0	0	0	0	0																																																																																								
1	2	3	2	0																																																																																								
3	4	1	5	0																																																																																								
2	6	3	3	0																																																																																								
2	5	3	1	0																																																																																								
0	0	0	0	1																																																																																								
0	1	2	1	0																																																																																								
2	3	0	4	0																																																																																								
1	5	2	2	0																																																																																								
1	4	2	0	0																																																																																								
<p>(iii)</p>	<table border="1" style="margin-left: 20px;"> <tr><td></td><td><i>C</i></td><td><i>D</i></td><td><i>L</i></td><td><i>S</i></td></tr> <tr><td><i>I</i></td><td>2</td><td>4</td><td>7</td><td>6</td></tr> <tr><td><i>J</i></td><td>4</td><td>6</td><td>5</td><td>9</td></tr> <tr><td><i>K</i></td><td>3</td><td>8</td><td>7</td><td>7</td></tr> <tr><td><i>N</i></td><td>3</td><td>7</td><td>7</td><td>5</td></tr> </table> <p>Reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>2</td><td>2</td><td>0</td><td>4</td></tr> <tr><td>1</td><td>4</td><td>2</td><td>2</td></tr> <tr><td>1</td><td>3</td><td>2</td><td>0</td></tr> </table> <p>Then reduce rows</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>2</td><td>1</td></tr> <tr><td>2</td><td>2</td><td>0</td><td>4</td></tr> <tr><td>0</td><td>3</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>3</td><td>2</td><td>0</td></tr> </table> <p>$I = D$ $J = L$ $K = C$ $N = S$</p>		<i>C</i>	<i>D</i>	<i>L</i>	<i>S</i>	<i>I</i>	2	4	7	6	<i>J</i>	4	6	5	9	<i>K</i>	3	8	7	7	<i>N</i>	3	7	7	5	0	0	2	1	2	2	0	4	1	4	2	2	1	3	2	0	0	0	2	1	2	2	0	4	0	3	1	1	1	3	2	0	<p>B1 This 4×4 matrix (need not have row and column labels)</p> <p>Or reduce rows</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>2</td><td>5</td><td>4</td></tr> <tr><td>0</td><td>2</td><td>1</td><td>5</td></tr> <tr><td>0</td><td>5</td><td>4</td><td>4</td></tr> <tr><td>0</td><td>4</td><td>4</td><td>2</td></tr> </table> <p>M1</p> <p>Then reduce columns</p> <table border="1" style="margin-left: 20px;"> <tr><td>0</td><td>0</td><td>4</td><td>2</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>3</td></tr> <tr><td>0</td><td>3</td><td>3</td><td>2</td></tr> <tr><td>0</td><td>2</td><td>3</td><td>0</td></tr> </table> <p>A1</p> <p>B1 This matching indicated in any way Correct answer only</p>	0	2	5	4	0	2	1	5	0	5	4	4	0	4	4	2	0	0	4	2	0	0	0	3	0	3	3	2	0	2	3	0	<p>[4]</p>
	<i>C</i>	<i>D</i>	<i>L</i>	<i>S</i>																																																																																								
<i>I</i>	2	4	7	6																																																																																								
<i>J</i>	4	6	5	9																																																																																								
<i>K</i>	3	8	7	7																																																																																								
<i>N</i>	3	7	7	5																																																																																								
0	0	2	1																																																																																									
2	2	0	4																																																																																									
1	4	2	2																																																																																									
1	3	2	0																																																																																									
0	0	2	1																																																																																									
2	2	0	4																																																																																									
0	3	1	1																																																																																									
1	3	2	0																																																																																									
0	2	5	4																																																																																									
0	2	1	5																																																																																									
0	5	4	4																																																																																									
0	4	4	2																																																																																									
0	0	4	2																																																																																									
0	0	0	3																																																																																									
0	3	3	2																																																																																									
0	2	3	0																																																																																									
Total = 18																																																																																												

2	(i)	-2	B1	Accept 'loses 2' or equivalent	[1]																																			
	(ii)	Column W is dominated by column Y . If Rowena plays P , Collette loses 2 with W but 1 with Y . If Rowena plays Q , Collette loses 1 with W but gains 1 with Y .	B1 B1	Stating Y (but not W dominates Y) Correct comparisons explained, $2 > 1$ and $1 > -1$, or equivalent	[2]																																			
	(iii)	<table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2"></th> <th colspan="4">Collette</th> <th></th> </tr> <tr> <th colspan="2"></th> <th>$[W]$</th> <th>X</th> <th>Y</th> <th>Z</th> <th>Row min</th> </tr> </thead> <tbody> <tr> <td>Rowena</td> <td>P</td> <td>$[2]$</td> <td>-3</td> <td>1</td> <td>3</td> <td>-3</td> </tr> <tr> <td></td> <td>Q</td> <td>$[1]$</td> <td>2</td> <td>-1</td> <td>-4</td> <td>-4</td> </tr> <tr> <td>Col max</td> <td></td> <td>$[2]$</td> <td>2</td> <td>1</td> <td>3</td> <td></td> </tr> </tbody> </table> <p>Play-safe for Rowena is P Play-safe for Collette is Y</p>			Collette							$[W]$	X	Y	Z	Row min	Rowena	P	$[2]$	-3	1	3	-3		Q	$[1]$	2	-1	-4	-4	Col max		$[2]$	2	1	3		M1 A1 A1	Determining row minima and column maxima, or equivalent. Must be correct, including W if shown. May not be implied from answers. P stated Y stated	[3]
		Collette																																						
		$[W]$	X	Y	Z	Row min																																		
Rowena	P	$[2]$	-3	1	3	-3																																		
	Q	$[1]$	2	-1	-4	-4																																		
Col max		$[2]$	2	1	3																																			
	(iv)	$-3p + 2(1-p) = 2-5p$ Y gives $2p-1$ Z gives $7p-4$	B1 B1	$2-5p$ in simplified form Both $2p-1$ and $7p-4$ in any form	[2]																																			
	(v)	 <p>$7p - 4 = 2 - 5p \Rightarrow p = 0.5$ $E = -0.5$</p>	B1 M1 A1 B1	Graph must be on graph paper Their lines drawn correctly on a reasonable scale Solving the correct pair of equations (only) or using graph correctly 0.5, correct answer only -0.5, correct answer only	[4]																																			
	(vi)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collette plays Y Rowena expects $4p_1 + 3p_2 + 6p_3$, and if Collette plays Z Rowena expects $7p_1 + 0p_2 + 2p_3$ We are solving a <u>maximin</u> problem. m is less than or equal to each of these values since we need find the maximum value of the worst possible augmented expected pay-off for each value of p	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns Y and Z respectively. Note: $4p_1 + 3p_2 + 6p_3$ and $7p_1 + 2p_3$ are given in question Or shown on a diagram. For <u>each value of p</u> we look at the minimum output.	[3]																																			
	(vii)	We use an inequality instead of an equality because this is needed to enable the Simplex algorithm to pivot on a row that will increase the value of M	B1	So that we can use the Simplex algorithm.	[1]																																			
	(viii)	$p_3 = \frac{3}{7}$ $E = \frac{6}{7}$	B1 B1	$\frac{3}{7}$ $\frac{6}{7}$	[2]																																			
Total = 18																																								

ANSWERED ON INSERT

3	(i)	$\{S, A, B, D, G\}, \{C, E, F, T\}$ (given) $AC = 4, BC = 2, BE = 1, DE = 2, GE = 5, GT = 6$ $4+2+1+2+5+6$ $= 20$ litres per minute	M1	Identifying the correct arcs, on a diagram or list or by using 4, 2, 1, 2, 5, 6	
			A1	20 from a correct calculation	[2]
	(ii)	At most 2 litres per minute can enter G so the arc GE can carry at most 2 litres per minute	B1	Maximum into $G = 2$	[1]
	(iii)	At most 8 litres per minute can flow into E Flow shown on diagram on insert Flow in = flow out for each vertex except S, T A feasible flow of 8 litres per minute through E	B1	8	[3]
			M1	A flow of the rate they have claimed through E (irrespective of whether it is feasible) (directions may not be changed, assume a blank means 0)	
			A1	No pipe capacities exceeded and flow through $E = 8$	
	(iv)	Arrows labelled on diagram $SA = 0 \quad AC = 0 \quad CF = 0 \quad FT = 1$ $AS = 4 \quad CA = 4 \quad FC = 4 \quad TF = 4$ $AB = 3 \quad BC = 2 \quad CE = 3 \quad EF = 4$ $BA = 0 \quad CB = 0 \quad EC = 0 \quad FE = 0$ $SB = 4 \quad BE = 0 \quad ET = 5$ $BS = 1 \quad EB = 1 \quad TE = 1$ $BD = 3 \quad DE = 2 \quad EG = 0$ $DB = 0 \quad ED = 0 \quad GE = 5$ $SD = 0 \quad DG = 0 \quad GT = 4$ $DS = 2 \quad GD = 2 \quad TG = 2$	M1	Assume blanks mean 0 Arrows on arcs on one of the routes $SACFT, SBET, SDGT$ labelled correctly, or all labels on the route reversed	[3]
			M1	Arrows on all three routes labelled correctly or all reversed	
			A1	All arrows labelled correctly, not reversed	
	(v)	Amount that flows along $SBDET = 2$ litres per min $SB = 4 \ 2 \quad BD = 3 \ 1 \quad DE = 2 \ 0 \quad ET = 5 \ 3$ $BS = 1 \ 3 \quad DB = 0 \ 2 \quad ED = 0 \ 2 \quad TE = 1 \ 3$	B1	2	[3]
			M1	For arrows on route $SBDET$: Labels updated consistently	
			A1	These all labelled correctly (and not reversed)	
	(vi)	Route used = $SBCET$ $SB = 4 \ 2 \ 0 \quad BC = 2 \ 0 \quad CE = 3 \ 1 \quad ET = 5 \ 3$ 1 $BS = 1 \ 3 \ 5 \quad CB = 0 \ 2 \quad EC = 0 \ 2 \quad TE = 1 \ 3$ 5	B1	$SBCET$ listed	[3]
			M1	For arrows on route $SBCET$: Labels updated consistently	
			A1	These all labelled correctly (and not reversed)	
	(vii)		B1	Follow through their (v) and (vi) if possible Assume blanks mean 0	[1]
	(viii)	Eg cut through arcs SA, SB, SD Or arcs AC, BC, BE, DE, DG	M1	A suitable cut chosen, indicated in any way	[2]
			A1	Indicated by listing arcs cut	
Total = 18					

PART (a) ANSWERED ON INSERT

4	(a)	Stage	State	Action	Working	Suboptimal maximum				
			0	0	5	5				
		2	1	0	4	4				
			2	0	4	4				
			0	0	$3 + 5 = 8$	8				
				1	$4 + 4 = 8$	8				
		1	1	1	$2 + 4 = 6$					
				2	$4 + 4 = 8$	8				
			2	1	$6 + 4 = 10$	10				
				2	$5 + 4 = 9$					
			0	0	$4 + 8 = 12$					
		0	0	1	$5 + 8 = 13$	13				
				2	$2 + 10 = 12$					
		Length of longest path = 13 Route = (0;0) – (1;1) – (2;2) – (3;0)								
							B1	5, 4, 4 identified as suboptimal maxima for stage 2		
							M1	Transferring suboptimal maxima from stage 2 to stage 1 correctly		
							A1	Correct additions or totals seen for all rows in stage 1		
							B1	8, 8, 10 identified as suboptimal maxima for stage 1 (cao)		
							M1	Transferring suboptimal maxima from stage 1 to stage 0 correctly		
							A1	Correct additions or totals seen for all rows in stage 0		
							B1	13		
							B1	Correct route or in reverse (including (0; 0) and (3; 0))	[8]	
	(b)(i)									
							M1	Condone directions missing Must be activity on arc A reasonable attempt, arcs should be labelled		
							A1	Any correct form Condone extra dummies provided precedences are not violated, accept networks with multiple end vertices Arc weights may be shown but are not necessary	[2]	
	(ii)	<p>Minimum project completion time = 13 days Critical activities B, G, L</p>								
							M1	Follow through their network if possible Values at vertices may be recorded using any consistent notation		
							A1	Forward pass with no more than one independent error		
							A1	Forward pass correct		
							M1	Backward pass with no more than one independent error (follow through their 13)		
							A1	Backward pass correct		
							B1	13 stated, cao		
							B1	B, G, L correct answer only	[6]	
	(iii)									
							B1	Not follow through A directed dummy from end of G to start of K		
							B1	A directed dummy from end of G to start of L Condone extra dummies provided precedences are not violated Watch out for K following I	[2]	
Total = 18										

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2008 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	63	55	47	39	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	56	49	43	37	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	41	35	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	47	38	29	20	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	56	47	38	29	21	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	59	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	26	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	54	47	40	34	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	33.3	50.4	65.4	77.0	86.6	100	14679
3891	100	100	100	100	100	100	1
3892	57.2	76.7	88.2	94.1	97.6	100	1647
7890	45.4	67.3	82.4	92.1	97.8	100	10512
7891	33.3	66.7	100	100	100	100	6
7892	56.5	77.9	90.0	95.4	98.2	100	1660

For a description of how UMS marks are calculated see:

http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity



OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553